Chapter 15 – Conditional Probability

Some basic probability terminology:

Independent Events
The outcome of one event does not affect the other

Mutually Exclusive (Disjoint) Events
Two events that do NOT have any outcomes in common (or two events that cannot both happen)

Some basic probability formulas… (these are on your formula chart)

Conditional Probability:

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

Probability of "A" or "B" (for events that are NOT mutually exclusive)

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

Probability Notation:

$P(A \cap B)$: Prob. A and B

$P(A \cup B)$: Prob A or B

$P(A|B)$: Prob. A given B

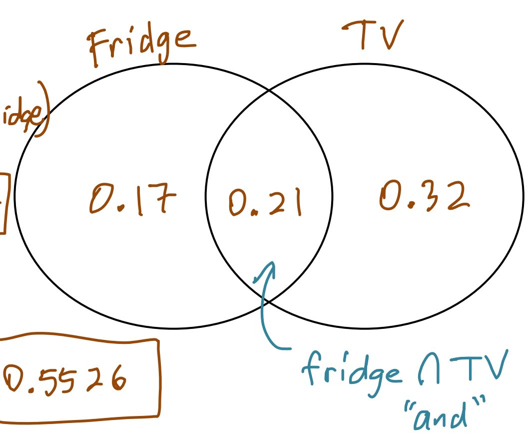
The Dorm Room Problem

A check of dorm rooms on a large college campus revealed that 38% had refrigerators, 53% had TVs, and 21% had both a TV and a refrigerator.

a) What is the probability that a randomly selected dorm room has a TV or a refrigerator?

$P(TV \cup fridge) = P(TV) + P(fridge) - P(TV \cap fridge)$ ("or")

$= 0.53 + 0.38 - 0.21 = \boxed{0.7}$



b) What is the probability that a dorm room with a refrigerator also has a TV?

$P(TV | fridge) = \frac{P(TV \cap fridge)}{P(fridge)} = \frac{0.21}{0.38} = \boxed{0.5526}$ ("given", "and")

c) What is the probability that a dorm room with a TV also has a refrigerator?

$P(fridge | TV) = \frac{0.21}{0.53} = \boxed{0.3962}$

d) Are the events "has a TV" and "has a refrigerator" mutually exclusive (aka, disjoint)? Explain.

No, they are NOT mutually exclusive – some dorm rooms have BOTH a TV and a refrigerator. If these events were mutually exclusive (disjoint), then it wouldn't be possible for both to occur at the same time.

The First Lady Problem

A Gallup survey of June 2004 asked U.S. adults who they think better fits their idea of what a first lady should be, Laura Bush or Hillary Rodham Clinton.

Response	18–29	30–49	50–64	Over 65	Total
Clinton	135	158	79	65	437
Bush	77	237	112	92	518
Equally/Neither/ No opinion	5	21	14	10	50
Total	**217**	**416**	**205**	**167**	**1005**

(Column heading: Age Group)

If we select a person at random from this sample:

a) What is the probability that the person thought Laura Bush best fits their first lady ideal?

$P(\text{Bush}) = \frac{518}{1005}$

b) What is the probability that the person is younger than 50 years old?

$P(<50) = \frac{217+416}{1005} = \frac{633}{1005}$

c) What is the probability that the person is younger than 50 *and* thinks Hillary Clinton best fits their ideal?

$P(<50 \cap \text{Clinton}) = \frac{135+158}{1005} = \frac{293}{1005}$

d) ☆ What is the probability that the person is younger than 50 *or* thinks Hillary Clinton best fits their ideal?

$P(<50 \cup \text{Clinton}) = P(<50) + P(\text{Clinton}) - P(<50 \cap \text{Clinton})$ (∪ = or)

$= \frac{633}{1005} + \frac{437}{1005} - \frac{293}{1005} = \frac{777}{1005}$

e) What is the probability that we choose a person between 18 and 29 who picked Clinton?

$P(18-29 \cap \text{Clinton}) = \frac{135}{1005}$

f) Among the 18-29 year olds, what is the probability that a person responded "Clinton"?

$P(\text{Clinton} \mid 18-29) = \frac{135}{217}$

g) What is the probability that a person who chose Clinton was between 18 and 29?

$P(18-29 \mid \text{Clinton}) = \frac{135}{437}$

h) Are the events **"thinks Hillary Clinton is the ideal first lady"** and **"is younger than 30"** independent? Justify your answer.

☆ Test for independence: $P(A) \overset{?}{=} P(A \mid B)$

$P(\text{Clinton}) \overset{?}{=} P(\text{Clinton} \mid 18-29)$

$\frac{437}{1005} \overset{?}{=} \frac{135}{217}$

$0.4348 \neq 0.6221$

No, "thinks Clinton is the ideal first lady" and "younger than 30" are NOT independent events -
P(Clinton) $\neq$ P(Clinton | 18-29)

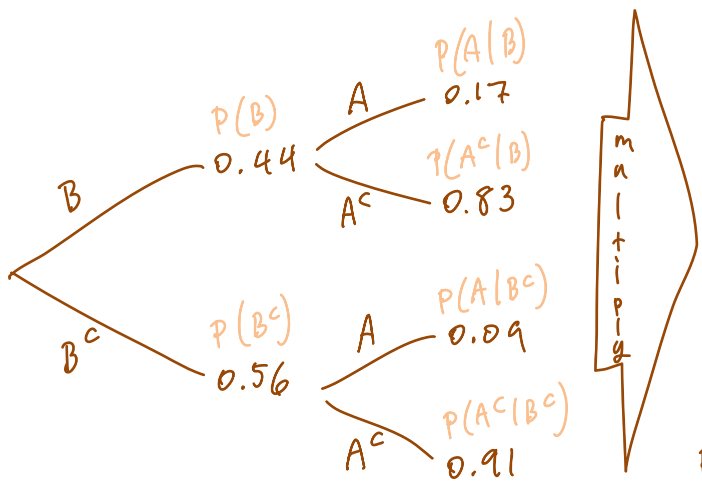
The Binge Drinking Problem

According to a study by the Harvard School of Public Health, 44% of college students engage in **binge drinking**. Another study finds that **among binge drinkers**, 17% have been involved in an **alcohol-related auto accident**, while only 9% of nonbingers of the same age have been involved in **such accidents**.

Make a tree diagram!

$B = \{\text{binge drinking}\}$

$A = \{\text{Alcohol-related accident}\}$



$P(B \cap A) = 0.0748$

$P(B \cap A^c) = 0.3652$

$P(B^c \cap A) = 0.0504$

$P(B^c \cap A^c) = 0.5096$

a) What is the probability that a college student is a binge drinker AND does not get into an alcohol-related accident?

$P(B \cap A^c) = \boxed{0.3652}$

b) What is the probability that a college student gets into an alcohol-related accident?

$P(A) = 0.0748 + 0.0504 = \boxed{0.1252}$

c) Among the students that are NOT binge drinkers, what proportion of students did NOT get into an alcohol-related accident?

$P(A^c | B^c) = \boxed{0.91}$

d) Among the students that ARE binge drinkers, what proportion of them DID get into an alcohol-related accident?

$P(A | B) = \boxed{0.17}$

e) *What is the probability that a student that got into an alcohol-related accident was a binge drinker?

$$P(B|A) = \frac{P(B \cap A)}{P(A)} = \frac{0.0748}{0.1252} = \boxed{0.5974}$$