

AP Statistics – Geometric and Binomial Probability Models

Two types of probability models for Bernoulli Trials:

I. Geometric Probability Model:

Number of attempts (trials) UNTIL 1st success

II. Binomial Probability Model:

Number of "successes" in a specified # of trials

3 conditions must be met for either of these:

- **Bi**: 2 outcomes (success/failure)
- Independent events
- Same Probability of success on each trial

The Geometric model – Geom(p)

p = probability of success

Formula(s): [not on the formula chart!]

X = # of trials UNTIL the 1st success

$$E(X) = \frac{1}{p}$$

(or μ_x)

1. The Hungarian Problem

On the "Hungarian Quiz" that we just took...
4 choices per question, guessing BLINDLY.

$$p = \frac{1}{4} \text{ or } 0.25$$

X = # of guesses UNTIL our 1st correct answer

- a) How many questions do you expect to answer until you get one correct?
(mean!) (GEOMETRIC!)

$$E(X) = \frac{1}{p} = \frac{1}{0.25} = \boxed{4}$$

but don't round if you get a decimal

- b) What's the probability that the first question you answer correctly is the 4th question?

1st success on attempt #4

$$P(X=4) = (0.75)(0.75)(0.75)(0.25) \leftarrow (3 \text{ wrongs... then a right})$$

or

$$(0.75)^3(0.25) = \boxed{0.1055}$$

- c) What is the probability that the first question you answer correctly is the 4th or 5th or 6th question? (eek)

$$\begin{aligned} P(X=4) &= (0.75^3)(0.25) = 0.1055 \\ \text{or} \\ P(X=5) &= (0.75^4)(0.25) = 0.0791 \\ \text{or} \\ P(X=6) &= (0.75^5)(0.25) = 0.0593 \end{aligned} \quad \left. \begin{array}{l} \\ \\ \end{array} \right\} \text{ADD } \boxed{0.2439}$$

The Binomial model - Binom(n, p)

n = number of trials

p = probability of "success"

X = # of successes

$q = 1 - p$ = prob. of "failure"

Formulas:

$$E(X) = np \quad \text{SD}(X) = \sqrt{np(1-p)} \quad P(X=k) = \binom{n}{k} p^k (1-p)^{n-k}$$

μ_x σ_x

2. The "Hungarian" Problem II

On that 10 question "Hungarian Quiz"...

X = # of correct answers

$n = 10$ $p = 0.25$ **BINOMIAL!**

a) What are the mean and standard deviation of the number of correctly answered questions?

$$E(X) = np = 10(0.25) = \boxed{2.5} \quad \text{SD}(X) = \sqrt{10(0.25)(0.75)} = \boxed{1.369...}$$

b) What is the probability that a student got exactly 4 questions correct?

(Hint: since we need to find the probability of getting ANY 4 questions correct - and since there are a number of ways for that to occur - we need to use a Binomial model here)

$$P(X=4) = \binom{10}{4} (0.25)^4 (0.75)^6 = \boxed{0.1460}$$

↑
10 nCr 4
on calculator

c) What is the probability that a student answered **no more than 5** correctly?

$$\begin{aligned} P(X \leq 5) &= P(X=0) + P(X=1) + P(X=2) + P(X=3) + P(X=4) + P(X=5) \\ &= \binom{10}{0}(0.25)^0(0.75)^{10} + \binom{10}{1}(0.25)^1(0.75)^9 + \dots + \binom{10}{5}(0.25)^5(0.75)^5 \\ &\quad \{ \text{USE "BINOMCDF"}(10, 0.25, 5) \} \\ &= \boxed{0.9803} \end{aligned}$$

d) What is the probability that a student answered **at least 1 question** correctly? (think back)

$$\begin{aligned} P(X \geq 1) &= 1 - P(X=0) \\ &= 1 - \binom{10}{0}(0.25)^0(0.75)^{10} \\ &= 1 - 0.0563 = \boxed{0.9437} \end{aligned}$$

e) What is the probability that a student answered **at least 4 questions** correctly? (ugh...)

$$\begin{aligned} P(X \geq 4) &= 1 - P(X \leq 3) \\ &= 1 - \text{BinomCDF}(\overset{n}{10}, \overset{p}{0.25}, \overset{\text{up to } X=3}{3}) \\ &= 1 - 0.7759 \\ &= \boxed{0.2241} \end{aligned}$$