AP Statistics
Random Variables Notes
(This is actually from the beginning of chapter 16, but this WILL be included on this test)

1) An insurance company offers a "death and disability" policy that pays $10,000 when the policy holder dies (which the company estimates will occur for 1 out of every 1000 people), or $5,000 when the policy holder is permanently disabled (estimated to occur for 2 out of every 1000 people). Based on actuarial information, the company has calculated the probabilities shown in the table below. The company plans to charge $50 per policy. Let the random variable "X" represent the **PROFIT** made by the insurance company per person.

Calculate and interpret the mean (expected value) and standard deviation of "X".

Outcome	Death	Disability	Neither
x	-$9950	-$4950	+$50
P(x)	0.001	0.002	0.997

MEAN:

μ_x or $E(x) = \sum x \cdot P(x)$

$= -9950(0.001) + -4950(0.002) + 50(0.997)$

$\boxed{E(x) = \$30}$

Interpretation: (this is a LONG-RUN average)

If the insurance company takes on a LARGE number of clients, their average PROFIT per client averages out to $30.

STANDARD DEVIATION (and variance)

$Var(x) = \sum (x-\mu)^2 \cdot P(x)$

$= (-9950-30)^2(0.001) + (-4950-30)^2(0.002) + (50-30)^2(0.997)$

$= 149600$ ← (technically, the units on variance are "DOLLARS SQUARED")

σ_x or $SD(x) = \sqrt{Var(x)}$

$= \sqrt{149600} = \boxed{\$386.78}$ ← For a LARGE number of clients, this is the typical/average(ish) difference from the mean profit.

2) Find the mean (expected value) and the standard deviation of the random variable "X".

x	60	70	80	90
P(x)	0.2	0.3	0.4	0.1

Answers: $E(x) = 74$

$SD(x) = 9.165...$

Scaling/Shifting with Means and Variances

remember: variance is (st. dev)^2

$E(X \pm c) =$ $E(X) \pm c$

$Var(X \pm c) =$ $Var(X)$

$SD(X \pm c) =$ $SD(X)$

$E(aX) =$ $a \cdot E(X)$

$Var(aX) =$ $a^2 \cdot Var(X)$

$SD(aX) =$ $a \cdot SD(X)$

For any two random variables, "X" and "Y":

$E(X \pm Y) =$ $E(X) \pm E(Y)$

If "X" and "Y" are independent:

ALWAYS a plus!

$Var(X \pm Y) =$ $Var(X) + Var(Y)$

$SD(X \pm Y) =$ $\sqrt{SD(X)^2 + SD(Y)^2}$

"X" and "Y" **MUST** be **independent!!!**
If they're NOT, then we cannot determine the variance (or standard deviation) of the combined random variable.

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important for SD on C, D, and F

3. X and Y are two independent random variables with the following attributes:

$E(X) = 11$	$E(Y) = 24$
$SD(X) = 9$	$SD(Y) = 5$

Find the mean and standard deviation of each of these random variables:

a) 3X $E(3X) = 3(11) = \boxed{33}$

$SD(3X) = 3 \cdot 9 = \boxed{27}$

b) Y − 15 $E(Y-15) = 24 - 15 = \boxed{9}$

$SD(Y-15) = \boxed{5}$ shift does NOT affect SD!

c) X + Y $E(X+Y) = 11 + 24 = \boxed{35}$

$SD(X+Y) = \sqrt{9^2 + 5^2} = \boxed{10.296}$

d) X − Y $E(X-Y) = 11 - 24 = \boxed{-13}$

$SD(X-Y) = \sqrt{9^2 + 5^2} = \boxed{10.296}$

ALWAYS ADD VARIANCE!

e) X1 + X2 + X3 (not the same as "3X"!!!)

$E(X_1 + X_2 + X_3) = 11 + 11 + 11 = \boxed{33}$

$SD(X_1 + X_2 + X_3) = \sqrt{9^2 + 9^2 + 9^2}$

$= \sqrt{3} \times 9 = \boxed{15.58}$

f) 5X − 3Y

$E(5X - 3Y) = 5(11) - 3(24)$

$= \boxed{-17}$

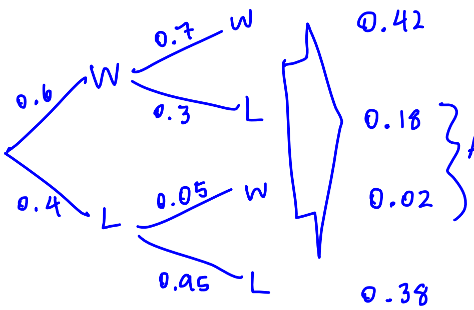
$SD(5X - 3Y) = \sqrt{(5 \times 9)^2 + (3 \times 5)^2}$

$= \boxed{47.43}$

4. **The Podunk Polar Bears** (a football team) have two games left in their season (so far they are winless). Experts estimate that the team has a 60% probability of winning the first game. If they win the first game, they have a 70% chance of winning the 2nd game. Otherwise, they only have a 5% chance of winning the second game.

make a table

Construct a probability model for the number of games that the Polar Bears will win.



x = # of wins	0	1	2
$P(x)$	0.38	0.20	0.42

The Die (Singular) Game Problem

5. You roll a die. If it comes up a 6, you win \$100.
If not, you get to roll again, and if you get a 6 the second time, you win \$50. If not, you lose ☹
Create a probability model for the amount you win at this game, and find the expected amount you'll win.

x = \$ won	100	50	0
$P(x)$	$\frac{1}{6}$	$\frac{5}{36}$	$\frac{25}{36}$
	roll a "6"	$\frac{5}{6}\cdot\frac{1}{6}$	$\frac{5}{6}\cdot\frac{5}{6}$

(if we round)

$E(x) = \$24 \qquad SD(x) = \$38$

Does "$X_1 + X_2$" = "$2X$"? (continuing the Die Game Problem…)

Find the mean and standard deviation of the amount of money won if…

a) we **double** the dollar amounts (and play the game once)

$E(2x) = 2(24) = \boxed{\$48}$

$SD(2x) = 2(38) = \boxed{\$76}$

b) we **play the game twice** (without doubling the \$ amounts)

$E(x_1 + x_2) = 24 + 24 = \boxed{\$48}$

$SD(x_1 + x_2) = \sqrt{38^2 + 38^2} = \boxed{\$54}$

Not the same!

c) we **play the game 200 times** (without changing the \$amounts)

$E(x_1 + x_2 + \dots + x_{200}) = 200(24) = \boxed{\$4800}$

$Var(x_1 + x_2 + \dots + x_{200}) = \underbrace{38^2 + 38^2 + \dots + 38^2}_{200\ times} = 200(38^2)$

$SD(x) = \sqrt{200(38^2)} = \boxed{\$537.4\ldots}$

6. The Matchmaker Problem I

In a far-away society, males and females are randomly selected to be matched up with each other for life ☺

Heights	Mean	SD
Males	**69.5**	**3.2**
Females	**65**	**2.8**

We will assume that

- *the heights for adult males and females are* ***independent***
- *the heights of both males and females are* ***approximately normally distributed***

a) Find the probability that the female is paired with a shorter man.

$M < F$ ... or $M - F < 0$

(This means male is shorter than female)

☆ First find μ and σ of $(M-F)$:

$E(M-F) = 69.5 - 65 = 4.5$

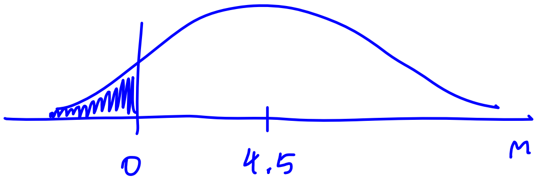
$SD(M-F) = \sqrt{3.2^2 + 2.8^2} = 4.252$

$P(M - F < 0)$

$= P(z < -1.0583)$

$= $ about $\boxed{0.1446}$

Using $N(4.5,\ 4.252)$:



$z = \frac{x - \mu}{\sigma}$

$= \frac{0 - 4.5}{4.252} = -1.0583$

7. The Matchmaker Problem II

b) Find the probability that the man is at least 12 inches taller than his lady.

WHOA, NELLY! this means

$M - F \geq 12$

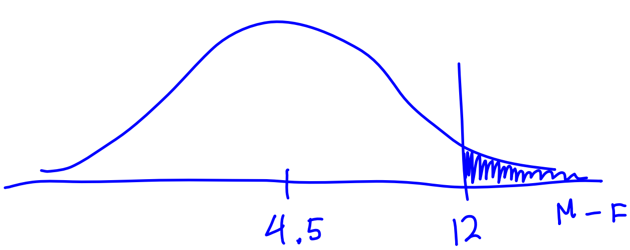
STILL USING

$N(4.5,\ 4.252)$ for "$M - F$":

$P(M - F \geq 12)$

$= P(z > 1.76)$

$=$ about $\boxed{0.0392}$



$z = \frac{x - \mu}{\sigma}$

$= \frac{12 - 4.5}{4.252}$

$z = 1.76\ldots$