AP Statistics – Geometric and Binomial Probability Models

Two types of probability models for Bernoulli Trials:

I. Geometric Probability Model:

Number of attempts (trials) UNTIL 1st success

II. Binomial Probability Model:

Number of "successes" in a specified # of trials

3 conditions must be met for either of these:

- **B**i : 2 outcomes (success/failure)
- **I**ndependent events
- **S**ame Probability of success on each trial

The Geometric model – Geom(p)

p = probability of success

X = # of trials UNTIL the 1st success

Formula(s): *[not on the formula chart!]*

$E(X) = \frac{1}{p}$ (or μ_X)

1. **The Hungarian Problem** On the "Hungarian Quiz" (4 choices per question, guessing BLINDLY.) that we just took…

$p = \frac{1}{4}$ or 0.25

X = # of guesses UNTIL our 1st correct answer

a) How many questions do you expect to answer until you get one correct? (mean!) (GEOMETRIC!)

$$E(X) = \frac{1}{p} = \frac{1}{0.25} = \boxed{4}$$

↑ but don't round if you get a decimal

b) What's the probability that the first question you answer correctly is the 4th question?

$P(X = 4)$ ← 1st success on attempt #4 $= (0.75)(0.75)(0.75)(0.25)$ ← (3 wrongs… then a right)

or

$$(0.75)^3(0.25) = \boxed{0.1055}$$

c) What is the probability that the first question you answer correctly is the 4th or 5th or 6th question? (eek)

$P(X = 4) = (0.75^3)(0.25) = 0.1055$

or

$P(X = 5) = (0.75)^4(0.25) = 0.0791$

or

$P(X = 6) = (0.75)^5(0.25) = 0.0593$

ADD $\boxed{0.2439}$

The Binomial model – Binom(n, p)

n = number of trials

p = probability of "success"

X = # of successes

$q = 1 - p$ = prob. of "failure"

Formulas:

$E(X) = np$ $\quad SD(X) = \sqrt{np(1-p)}$ $\quad P(X=k) = \binom{n}{k} p^k (1-p)^{n-k}$

μ_X $\quad \sigma_X$

2. The "Hungarian" Problem II

On that 10 question "Hungarian Quiz"... → X = # of correct answers

$n = 10$ $\quad p = 0.25$ $\quad$ BINOMIAL!

a) What are the mean and standard deviation of the number of correctly answered questions?

$E(X) = np = 10(0.25) = \boxed{2.5}$ $\quad SD(X) = \sqrt{10(0.25)(0.75)} = \boxed{1.369\ldots}$

b) What is the probability that a student got exactly 4 questions correct?
(Hint: since we need to find the probability of getting ANY 4 questions correct – and since there are a number of ways for that to occur – we need to use a Binomial model here)

$P(X=4) = \binom{10}{4}(0.25)^4(0.75)^6 = \boxed{0.1460}$

↑ 10 nCr 4 on calculator

c) What is the probability that a student answered **no more than 5** correctly?

$P(X \le 5) = P(X=0) + P(X=1) + P(X=2) + P(X=3) + P(X=4) + P(X=5)$

$= \binom{10}{0}(0.25)^0(0.75)^{10} + \binom{10}{1}(0.25)^1(0.75)^9 + \ldots + \binom{10}{5}(0.25)^5(0.75)^5$

{USE "BINOMCDF (10, 0.25, 5)}

$= \boxed{0.9803}$

d) What is the probability that a student answered **at least 1 question** correctly? *(think back)*

$P(X \ge 1) = 1 - P(X=0)$

$= 1 - \binom{10}{0}(0.25)^0(0.75)^{10}$

$= 1 - 0.0563 = \boxed{0.9437}$

e) What is the probability that a student answered **at least 4 questions** correctly? *(ugh...)*

$P(X \ge 4) = P(X=4) + P(X=5) + \cdots + P(X=10)$

$= \binom{10}{4}(0.25)^4(0.75)^6 + \ldots + \binom{10}{10}(0.25)^{10}(0.75)^0$

$= \boxed{0.2241}$

On the calculator:

$1 - [\text{BinomCDF}(10, 0.25, 3)$ (10 = n, 0.25 = p, up to x=3)

$1 - 0.7759$