AP STATISTICS **Name: _____ Period: ____**

Review Unit VI – Probability Models and Sampling Distributions

Show all work and reasoning.

1. Professional football players in the NFL have a distribution of salaries that is unimodal and heavily skewed to the larger salaries - the mean is $1.9 million, however this is distorted by a small number of players who made an insanely large amount of money (such as Peyton Manning, who in 2014 earned $15 million). In fact, the median salary for NFL players in 2014 was "only" $770,000. The standard deviation for NFL salaries is $2.34 million.

   a) Suppose you randomly select 150 players from the NFL and created a dot plot of their salaries. What is the likely shape of the distribution of those 150 salaries?

   b) Now suppose that you repeatedly select samples (with replacement) of 150 NFL players, and for each sample, you record the mean salary of the 150 players selected. If you repeated this procedure for a large number of samples, what is the likely shape of the distribution of mean salaries for random samples of 150?

   c) Find the mean and standard deviation for the distribution of the sample mean salaries for random samples of 150 players.

2. An assembly machine creates spherical widgets with a mean weight $\mu = 320$ mm. The machine has some variability, so the standard deviation of the diameters of each widget is $\sigma = 0.024$ mm. The machine operator inspects a random sample of widgets each hour for quality control purposes and records the sample mean diameter $\bar{x}$. How many widgets would you need to sample if you wanted the standard deviation of the sampling distribution of $\bar{x}$ to be 0.002 mm? Justify your answer (show your work).

3. Podunk Research reports that this past Thanksgiving, the mean weight for Thanksgiving turkeys was 30 pounds. Assuming that Thanksgiving turkey weights are approximately normally distributed, which of the following is MORE likely? Justify your answer. *(You may wish to use drawings/diagrams to help justify your answer)*

   i. A random sample of 15 turkeys having a mean weight that is greater than 34 pounds

   or

   ii. A random sample of 50 turkeys having a mean weight that is greater than 34 pounds

4. If $p = 0.24$, what is the smallest value of n for which the distribution of $\hat{p}$ will be well-approximated by a normal distribution?

5. If $p = 0.90$, what is the smallest value of n for which the distribution of $\hat{p}$ will be well-approximated by a normal distribution?

6. What would you need to multiply the sample size by in order to cut the standard deviation of the sampling distribution to...

   a) half of its original size?

   b) one-third of its original size?

   c) one-tenth of its original size?

7. Each of the faces of a fair six-sided number cube is numbered with one of the numbers 1 through 6, with a different number appearing on each face. Two such number cubes – one orange and one white – will be tossed, and the sum of the numbers appearing on the faces that land up will be recorded.
   (Hint: Consider making a table of every possible combination of outcomes for 2 dice…)

   a) What is/are the most likely sum(s)? With what probability?

   b) What is/are the least likely sum(s)? With what probability?

8. Cutting-edge scientific research suggests that the true population proportion of high school seniors that develop the horrible disease known as **SENIORITIS** is 44%. Mr. Youn, however, believes* that this proportion is much higher for AP Statistics students **(BUT NOT YOU GUYS OF COURSE… COUGH, COUGH)**.

   a) Let's assume that the 44% estimate is indeed the truth. Describe the distribution for the sample proportion of seniors that develop senioritis for random samples of 158 students.
   *Hint: you need to describe the **shape** (approximately normal? and why?), **center** (mean), and **spread** (standard deviation).*

   b) Suppose we take a random sample of 158 seniors that are taking AP Statistics (which, for the purposes of this problem, we will consider to be a representative sample of high school seniors). Using the information calculated in part (a), what is the probability that at least 50% of that sample will develop **SENIORITIS**?
   Be sure to verify any conditions that are necessary for your analysis.

** please do NOT feel compelled to prove us right. -_-*

9. The distribution of scores for persons over 26 years of age on the Wechsler Adult Intelligence Scale (WAIS) is approximately symmetric and unimodal with mean 100 and standard deviation of 15. The WAIS is one of the most common "IQ" tests for adults.

   a) Suppose we take a random sample of 60 adults. Describe the distribution of sample mean scores for random samples of 60 adults. *(Again, you need to describe the shape, mean, and standard deviation)*

   b) What is the probability that the average WAIS score of a *random sample* of 60 adults who take the WAIS test is 102 or higher? *Be sure to verify any conditions that are necessary for your analysis.*

10. A company ships gift baskets that contain apples and pears. The distributions of weight for the apples, the pears, and the baskets are each approximately normal. The mean and standard deviation for each distribution is shown in the table below. The weights of the items are assumed to be independent.

Item	Mean	Standard Deviation
Apple	4.7 ounces	0.28 ounce
Pear	5.4 ounces	0.21 ounce
Basket	10.9 ounces	1.65 ounces

a) Let the random variable W represent the total weight of 4 apples, 8 pears, and 1 basket. Describe the distribution of W. *(You probably guessed it: shape, center, and spread)*

b) At the quality assurance center, these gift baskets are discarded if the weight of the total basket is less than 67 ounces. Find the probability that a gift basket (consisting of 4 apples, 8 pears, and 1 basket) is discarded.

11. An insurance company estimates that the mean annual profit of each homeowner's policy written is \$150, with a standard deviation of \$6000. Let the random variable "X" represent the annual profit of each policy. Assume that each policy is independent of one another.

    a) If they write 7,000 of these policies ($X_1 + X_2 + \ldots + X_{7000}$), what are the mean and standard deviation of the total annual profit?

    b) Company executives will give their employees a bonus if their total annual profit exceeds \$2 million. What is the probability that this occurs?

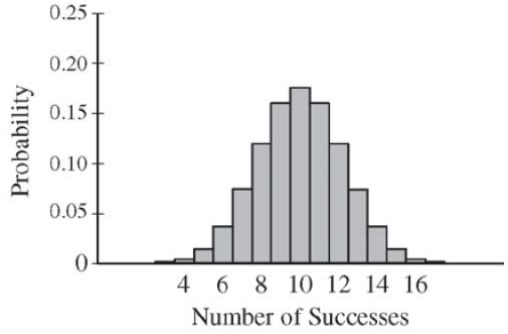
    c) Based on your answer to part (b), would you be surprised if the company employees earned a bonus? Explain.

12. _____ Let X be a random variable that has a skewed distribution with mean $\mu = 10$ and standard deviation $\sigma = 10$. Based on random samples of size 400, the sampling distribution of $\bar{x}$ is

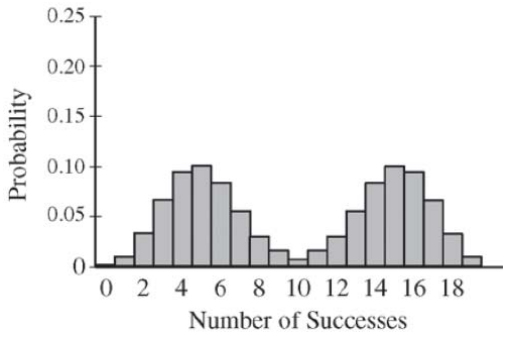
A) approximately normal with mean 10 and standard deviation 0.5
B) approximately normal with mean 10 and standard deviation 10
C) highly skewed with mean 10 and standard deviation 10
D) highly skewed with mean 10 and standard deviation 5
E) highly skewed with mean 10 and standard deviation 0.5

13. _____ Which of the following graphs represents a binomial distribution with $n = 20$ and $p = 0.75$?

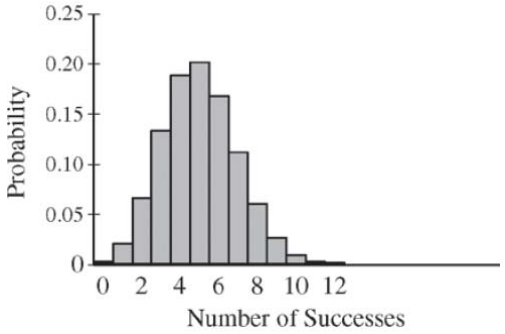
(A)



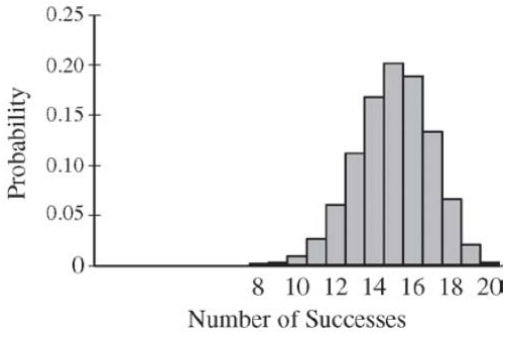
(B)



(C)



(D)



14. _____ In the design of a survey, which of the following best explains how to reduce sampling variability?

A) Increase the sample size.
B) Decrease the sample size.
C) Randomly select the sample.
D) Increase the number of questions in the survey.
E) Carefully word and field-test survey questions.

15. _____ Which of the following pairs of sample size n and population proportion p would produce the greatest standard deviation for the sampling distribution of a sample proportion $\hat{p}$?

A) $n = 500$ and p close to 0
B) $n = 500$ and p close to 1
C) $n = 500$ and p close to 0.5
D) $n = 200$ and p close to 0
E) $n = 200$ and p close to 0.5

16. _____ A researcher plans a study to examine the depth of belief in God among the adult population. He obtains a simple random sample of 100 adults as they leave church one Sunday morning. All but one of them agrees to participate in the survey. Which of the following are true statements?

I. Proper use of chance as evidenced by the simple random sample makes this a well-designed survey.
II. The high response rate makes this a well-designed survey.
III. Selection bias makes this a poorly designed survey.

A) I only
B) II only
C) III only
D) I and II
E) None of these

17. _____ A population of Jelly Blubbers has a mean length of 19.4 millimeters. A group of statistics students tags each blubber with a number, then uses a random number generator to select a random sample of 10 blubbers. The mean size observed in this sample of blubbers was 20.9 mm. Assuming that these students knew how to properly use a random number generator, and that the measurements were collected properly, what is the explanation for this difference between the observed mean and the expected mean?

A) Bias
B) Sampling variability
C) Confounding
D) Placebo effect
E) Extrapolation

18. _____ A professional sports team evaluates potential players for a certain position, using speed in the 40 yard dash as one of the main characteristics. In a random sample of 75 players, the times to run the 40 yards have a mean of 4.60 seconds and a standard deviation of 0.15 seconds. The minimum time of these 75 players was 4.40 seconds. Which of the following is most likely true about the 40 yard times for the players in this sample?

A) The distribution of times is approximately normal because the sample size is 75, and therefore the central limit theorem applies.
B) The distribution of times is approximately normal because the standard deviation is less than the mean.
C) The distribution of times is skewed to the right because the least possible time is within 2 standard deviations of the mean.
D) The distribution of times is skewed to the left because the least possible time is within 2 standard deviations of the mean.
E) The distribution of times has a median that is greater than the mean.

19. _____ Which of the following does simple random sampling help to reduce?

A) Bias resulting from poorly worded questions.
B) Bias resulting from undercoverage and nonresponse.
C) Bias resulting from the behavior of the interviewer.
D) None of the above.

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ANSWERS

1. a) Skewed to the right (this is asking about a SINGLE sample… not the distribution of means for a large number of samples)
   b) Approximately normal
   c) Mean = $1.9 million, SD = $0.191 million

2. 144 widgets

3. (i) is more likely. A smaller sample size results in a smaller z-score for the observed mean weight…

4. About 42

5. 100

6. a) x4
   b) x9
   c) x100

7. a) 7. Probability of 6/36
   b) 2 or 12. Each have a 1/36 probability.

8. a) Approximately normal, mean = 0.44, SD = 0.0395
   b) 0.0643

9. a) Approximately normal, mean = 100, SD = 1.936
   b) 0.1508

10. a) Approximately normal, E(W) = 72.9 ounces, SD(W) = 1.84
    b) 0.0007

11. a) Mean = $1,050,000; SD = $501,996
    b) 0.0294
    c) Yes (probability < 5%)

12. A
13. D
14. A
15. E

16. C
17. B
18. C
19. D