Inference with Means
(one sample)

AP Statistics
Chapter 23

William Gossett
1876 - 1937
Guinness employee

Dublin, Ireland

"Quality Assurance" (a.k.a., taste tester) for **Guinness beer**
(HORRIBLE, JOB, RIGHT?)

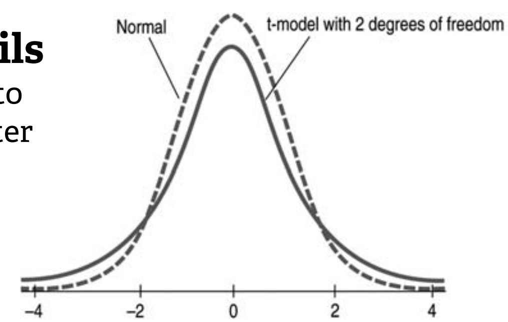
William Gossett

- Taste-tested batches of the dark ale in samples
- Calculated that he should reject good beer about 5% of the time
- Actually rejected good beer about 15% of the time **(WHAT?!)**
- Found that the Normal model doesn't play nice with small samples…

What Does This *Mean* for *Means*?

The Student's *t-models*

These curves have **more area in the tails**
(Because we are using "s" to estimate "σ", there is greater "uncertainty"/variability)

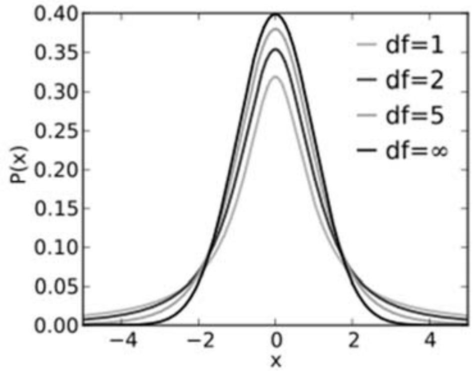


What Does This Mean for Means?

The Student's *t-models*

***Degrees of freedom** (df)*

Each sample size has its own model/curve!



For 1-sample means:

df = n – 1

Student's t-models

(you want to write this stuff down…)

- Use when we **don't know the population standard deviation σ** *(when we use the sample standard deviation as an estimate for σ)*
- t-distribution has fatter tails than z-distribution
- As df increase, the t-models look more and more like the Normal model.
- In fact, the t-model with **infinite** degrees of freedom is **exactly** Normal.

Calculating t vs z

(you want to write this down too…)

If we know σ:
(population SD)

$$z = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}}$$

If we only know s:
(sample SD)

$$t = \frac{\bar{x} - \mu}{s / \sqrt{n}}$$

t-table Practice

Find the critical value of *t* for 95% confidence with…

a) df = 10

t* = 2.228

b) n = 20

t* = 2.093

(use df = 20 – 1 = 19)

c) df = 32

t* = 2.042

(use df = 30 – round DOWN)

able entry for p
and C is the point
t* with probability
p lying above it
and probability C
lying between
−t* and t*.

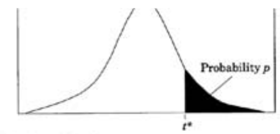


Table B — *t* distribution critical values

df	Tail probability *p* .25	.20	.15	.10	.05	.025	.02	.01	.005	.0025	.001	.0005
1	1.000	1.376	1.963	3.078	6.314	12.71	15.89	31.82	63.66	127.3	318.3	636.6
2	.816	1.061	1.386	1.886	2.920	4.303	4.849	6.965	9.925	14.09	22.33	31.60
3	.765	.978	1.250	1.638	2.353	3.182	3.482	4.541	5.841	7.453	10.21	12.92
4	.741	.941	1.190	1.533	2.132	2.776	2.999	3.747	4.604	5.598	7.173	8.610
5	.727	.920	1.156	1.476	2.015	2.571	2.757	3.365	4.032	4.773	5.893	6.869
6	.718	.906	1.134	1.440	1.943	2.447	2.612	3.143	3.707	4.317	5.208	5.959
7	.711	.896	1.119	1.415	1.895	2.365	2.517	2.998	3.499	4.029	4.785	5.408
8	.706	.889	1.108	1.397	1.860	2.306	2.449	2.896	3.355	3.833	4.501	5.041
9	.703	.883	1.100	1.383	1.833	2.262	2.398	2.821	3.250	3.690	4.297	4.781
10	.700	.879	1.093	1.372	1.812	2.228	2.359	2.764	3.169	3.581	4.144	4.587
11	.697	.876	1.088	1.363	1.796	2.201	2.328	2.718	3.106	3.497	4.025	4.437
12	.695	.873	1.083	1.356	1.782	2.179	2.303	2.681	3.055	3.428	3.930	4.318
13	.694	.870	1.079	1.350	1.771	2.160	2.282	2.650	3.012	3.372	3.852	4.221
14	.692	.868	1.076	1.345	1.761	2.145	2.264	2.624	2.977	3.326	3.787	4.140
15	.691	.866	1.074	1.341	1.753	2.131	2.249	2.602	2.947	3.286	3.733	4.073
16	.690	.865	1.071	1.337	1.746	2.120	2.235	2.583	2.921	3.252	3.686	4.015
17	.689	.863	1.069	1.333	1.740	2.110	2.224	2.567	2.898	3.222	3.646	3.965
18	.688	.862	1.067	1.330	1.734	2.101	2.214	2.552	2.878	3.197	3.611	3.922
19	.688	.861	1.066	1.328	1.729	2.093	2.205	2.539	2.861	3.174	3.579	3.883
20	.687	.860	1.064	1.325	1.725	2.086	2.197	2.528	2.845	3.153	3.552	3.850
21	.686	.859	1.063	1.323	1.721	2.080	2.189	2.518	2.831	3.135	3.527	3.819
22	.686	.858	1.061	1.321	1.717	2.074	2.183	2.508	2.819	3.119	3.505	3.792
23	.685	.858	1.060	1.319	1.714	2.069	2.177	2.500	2.807	3.104	3.485	3.768
24	.685	.857	1.059	1.318	1.711	2.064	2.172	2.492	2.797	3.091	3.467	3.745
25	.684	.856	1.058	1.316	1.708	2.060	2.167	2.485	2.787	3.078	3.450	3.725
26	.684	.856	1.058	1.315	1.706	2.056	2.162	2.479	2.779	3.067	3.435	3.707
27	.684	.855	1.057	1.314	1.703	2.052	2.158	2.473	2.771	3.057	3.421	3.690
28	.683	.855	1.056	1.313	1.701	2.048	2.154	2.467	2.763	3.047	3.408	3.674
29	.683	.854	1.055	1.311	1.699	2.045	2.150	2.462	2.756	3.038	3.396	3.659
30	.683	.854	1.055	1.310	1.697	2.042	2.147	2.457	2.750	3.030	3.385	3.646
40	.681	.851	1.050	1.303	1.684	2.021	2.123	2.423	2.704	2.971	3.307	3.551
50	.679	.849	1.047	1.299	1.676	2.009	2.109	2.403	2.678	2.937	3.261	3.496
60	.679	.848	1.045	1.296	1.671	2.000	2.099	2.390	2.660	2.915	3.232	3.460
80	.678	.846	1.043	1.292	1.664	1.990	2.088	2.374	2.639	2.887	3.195	3.416
100	.677	.845	1.042	1.290	1.660	1.984	2.081	2.364	2.626	2.871	3.174	3.390
1000	.675	.842	1.037	1.282	1.646	1.962	2.056	2.330	2.581	2.813	3.098	3.300
∞	.674	.841	1.036	1.282	1.645	1.960	2.054	2.326	2.576	2.807	3.091	3.291
	50%	60%	70%	80%	90%	95%	96%	98%	99%	99.5%	99.8%	99.9%

Confidence level *C*

conditions for *inference with 1-sample means*

1. **Randomization**
   Have an SRS or representative of population

2. **10% Condition**
   This is not as important to check for means, as sample sizes are usually very small, but it never hurts to be safe.

3. **Nearly Normal Condition**
   Check for <u>one</u> of the following:
   1. Given that population is roughly normally distributed
   2. Large enough sample size (n > 30) – CLT
   3. Check graph (**dotplot** is easy to make by hand) and check for plausible normality

   WATCH OUT FOR OUTLIERS!!!

Our first problem with real "data"

Let's pretend that the Chip's Ahoy company claims a mean of 24 chips per cookie… do we have statistical evidence (at α = 0.05) to doubt them?

Suppose we take a random sample of 10 of the company's cookies, and get the following counts for # of chocolate chips:

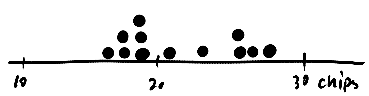
21	28	19	19	23	18
18	19	26	17	26	27

One-sample *t*-**test**

μ = true mean # of chips per cookie

Ho: μ = 24
Ha: μ < 24
(because we probably wouldn't be concerned if we got MORE than 24 chips per cookie)

Conditions:
- We have a random sample of the company's cookies
- *(Since n < 30, we must make a graph...)*
  The graph of sample data shows no outliers, so **normality should be plausible for the population**.



One-sample *t*-**test**

μ = true mean # of chips per cookie

Ho: μ = 24
Ha: μ < 24

$$t = \frac{\bar{x} - \mu}{s/\sqrt{n}} = \frac{21.75 - 24}{4.025/\sqrt{12}}$$

(fill in numbers in the formula, then just use the "t" and "p-value" from calculator)

t = -1.93 p = 0.0394 df = 11

Since p < alpha, we reject Ho. We HAVE evidence that the mean # of chips per cookie is less than 24.

Estimate the mean number of chocolate chips per cookie by using a 90% confidence interval

One-sample *t*-**interval**

$$\bar{x} \pm t^{*}_{df} \times \frac{s}{\sqrt{n}}$$

$$= 21.75 \pm (1.796) \times {}^{4.025}\!/_{\sqrt{12}}$$

(again, fill in numbers in the formula, then just get the interval from the calculator)

(19.633, 23.837)

We are 90% confident that the true MEAN number of chips per cookie is between 19.633 and 23.837.