

AP STATISTICS

P-values and Power Practice

Please work these out on separate paper.

1. **Error probabilities** You read that a statistical test at significance level $\alpha = 0.05$ has power 0.78. What are the probabilities of Type I and Type II errors for this test?
2. **Error probabilities** You read that a statistical test at the $\alpha = 0.01$ level has probability 0.14 of making a Type II error when a specific alternative is true. What is the power of the test against this alternative?
3. **Power** A drug manufacturer claims that fewer than 10% of patients who take its new drug for treating Alzheimer's disease will experience nausea. To test this claim, a significance test is carried out of
 $H_0: p = 0.10$ $H_A: p < 0.10$
You learn that the power of this test at the 5% significance level against the alternative $p = 0.08$ is 0.64.
 - a) Explain in simple language what "power = 0.64" means in this setting.
 - b) You could get higher power against the same alternative with the same α by increasing the number of measurements you make (a.k.a., increasing the sample size). Explain how this would increase the power of the test.
 - c) If you decide to use $\alpha = 0.01$ in place of $\alpha = 0.05$, with no other changes in the test, will the power increase or decrease? Justify your answer.
4. **Luke's Lab Partner(s)** For his statistics project, Luke is performing a hypothesis test to compare the proportion of people in Austin that have tattoos to the proportion in the rest of the United States (the true proportion of U.S. adults with a tattoo is believed to be 21%, according to a recent Harris Poll). Based on a survey that Luke conducts in downtown Austin, he calculates a z-statistic of +1.40.
(Hint: use a z-table/normal model and shade in the direction of the alternative hypothesis)
 - a) Since Luke believes Austin to be "weird" and thinks that the proportion of Austin adults with a tattoo will be greater than that for the rest of the country, his alternative hypothesis is " $H_A: p > 0.21$ ". Based on the z-score that he calculated and his alternative hypothesis, calculate the p-value for Luke's hypothesis test.
 - b) Luke's lab partner, Ciara, thinks that Austin adults will have a lower proportion of tattoos than the rest of the States, so she sets her alternative hypothesis as " $H_A: p < 0.21$ ". Based on the same z-score of 1.40 (since she used the same data as Luke), calculate the p-value for Ciara's hypothesis test.
 - c) Marie is a friend of Luke and Ciara, and while she is not even taking Statistics, decides to join in on the fun. Marie decides to use the same data to perform a two-tailed test (so her alternative hypothesis is " $p \neq 0.21$ "). Using the same z-score as Luke and Ciara, calculate the p-value for Marie's hypothesis test.
5. **More Power** Remember that the power of a test is defined as the probability of rejecting a false null hypothesis. Suppose that the null hypothesis of a test is $H_0: p = 0.50$.
(HINT: we are MOST likely to reject a false H_0 – and thus get higher power – with an actual proportion that agrees strongly with the H_A)
 - i) Suppose $H_A: p > 0.50$, and that H_A is true. For a fixed sample size and significance level α , the power of the test will be greatest if the actual proportion is which of the following?
a) 0.40 b) 0.48 c) 0.52 d) 0.60 e) 0.64
 - ii) Suppose $H_A: p < 0.50$, and that H_A is true. For a fixed sample size and significance level α , the power of the test will be greatest if the actual proportion is which of the following?
a) 0.40 b) 0.48 c) 0.52 d) 0.60 e) 0.64
 - iii) Suppose $H_A: p \neq 0.50$, and that H_A is true. For a fixed sample size and significance level α , the power of the test will be greatest if the actual proportion is which of the following?
a) 0.40 b) 0.48 c) 0.52 d) 0.60 e) 0.64