**Show all work (BUT ONLY IF YOU WANT CREDIT). All normal model calculations should include a well-labeled, shaded diagram.**

1. What are the mean, standard deviation, and total area under the curve in the standard normal model?

2. Brandy scored a 680 on Test A with a mean of 500 and a standard deviation of 100. Later on Test B which has a mean of 142 and a standard deviation of 16, Brandy earned a 170. Which test did she score higher on relative to the rest of the test takers?

3. An employee for a tech company has a salary which, when compared to all other employees at the company, has a z-score of 1.2. Interpret the meaning of this value in context. Also, if another employee has a salary which has a z-score of –2.3, what would that value mean?

4. _____ The adult lengths of a particular species of fish are normally distributed with a mean of 50 cm and a standard deviation of 10 cm. Which of the following intervals contains the largest proportion of adult lengths for this species of fish? *(Hint: Try to reason your way through this problem WITHOUT performing any calculations!)*

    A) 30 cm to 50 cm   B) 40 cm to 60 cm   C) 50 cm to 70 cm   D) 60 cm to 80 cm
    E) Each of these intervals contain the same proportion of adult lengths for this fish species.

5. _____ A distribution has a mean of 9 and a standard deviation of 4. If we must translate the summary statistics using the linear function $f(x) = 3x + 5$, the new mean and standard deviation will be

    A) 32, 17   B) 9 , 4   C) 14, 12   D) 32, 12
   E) we must first know the units of the data

6. _____ The amount of time required for each of 100 mice to navigate through a maze was recorded. The histogram below shows the distribution of times, in seconds, for the 100 mice.
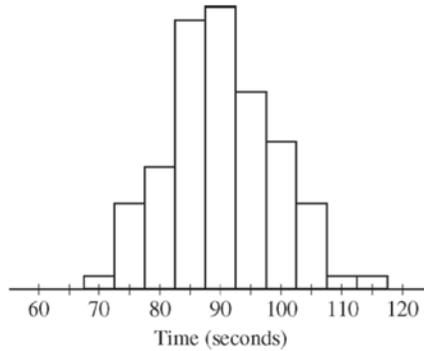
Time (seconds)

Which of the following values is closest to the standard deviation of the 100 times?

A) 2.5 seconds   B) 10 seconds   C) 20 seconds   D) 50 seconds   E) 90 seconds

6b. Based on your answer to #6, what is the variance for the distribution of navigation times?

7. _____ The histogram to the right shows the distribution of salaries for professional basketball players in the National Basketball Association (NBA) in 2006. Many players make large salaries and a small handful make ASTRONOMICAL salaries. The mean salary for this population is about $3.7 million, with a standard deviation of about $4 million. At the time*, Lebron James was earning an annual salary of about $5.9 million. Which of the following statements is/are valid?
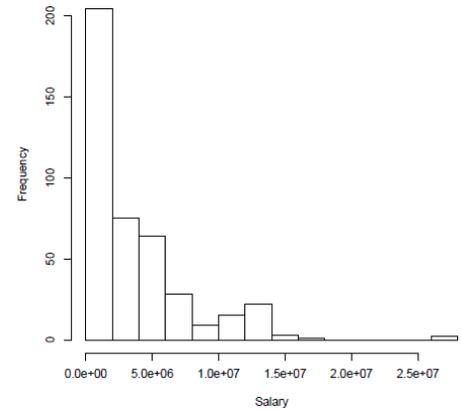
Salary

I. The z-score for Lebron James' salary is about z = 0.55.
II. Approximately 70.88% of NBA players in 2006 earned a lower annual NBA salary than Lebron James.
III. This distribution of salaries would be better summarized by median and IQR rather than by mean and standard deviation.

A) I only   B) II only   C) III only   D) I and II
E) I and III   H) II and III   J) All are valid   K) None are valid

8. _____ The distribution of the diameter of pipes from a manufacturer follows an approximately normal distribution. Based on a very large sample, it was found that 20 percent of the pipes were less than 9.667 inches in diameter, and thirty percent of the pipes were above 10.578 inches in diameter. What are the mean and standard deviation of the distribution of diameters of the pipes?

A) $\mu = 10.56$, $\sigma = 0.65$
B) $\mu = 10.61$, $\sigma = 0.55$
C) $\mu = 10.23$, $\sigma = 0.67$
D) $\mu = 11.50$, $\sigma = 0.65$
E) $\mu = 11.21$, $\sigma = 0.67$

9._____ Rudy measured the lengths of cords to be worn by the honors frisbee students at Hipster Sports High. He found the distribution of lengths to be approximately normally distributed with a mean of 80 inches and a standard deviation of 0.75 inches. The principal wanted the measurements to be in feet. What would be the mean and standard deviation in feet?

A) 960, 9   B) 6.667, 0.0625   C) 6.667, 0.75   D) 80, 0.75
E) Rudy must measure again in feet

10. _____ Populations P1 and P2 are normally distributed and have identical means. However, the standard deviation of P1 is twice the standard deviation of P2. What can be said about the percentage of observations falling within two standard deviations of the mean for each population?

A) The percentage for P1 is twice the percentage for P2.
B) The percentage for P1 is greater, but not twice as great, as the percentage for P2.
C) The percentage for P2 is twice the percentage for P1.
D) The percentages are identical.
E) The percentage for P2 is greater, but not twice as great, as the percentage for P1.

11. _____ A factory dumps an average of 2.43 tons of pollutants into a river every week. If the standard deviation is 0.88 tons and the distribution is approximately normal, what proportion of weeks does the factory dump more than 3 tons of pollutants in the river?

A) 0.2578   B) 0.2843   C) 0.6500   D) 0.7157   E) 0.7422

12. _____ The noise level in a restaurant is normally distributed with a mean of 30 decibels and a standard deviation of 4 decibels. Ninety-nine percent of the time the noise level is below what level?

A) 20.7   B) 32.0   C) 33.4   D) 37.8   E) 39.3

13. _____ Jay Olshansky from the University of Chicago was quoted in *Chance News* as arguing that for the average life expectancy to reach 100, 18% of people would have to live to 120. What standard deviation is he assuming for this statement to make sense? (Assume that the distribution for life expectancy is normally distributed)

A) 21.7   B) 24.4   C) 25.2   D) 35.0   E) 111.1

14. _____ A coffee machine can be adjusted to deliver any fixed number of ounces of coffee. If the amount of coffee delivered is roughly normally distributed, and the machine has a standard deviation in delivery equal to 0.4 ounces, what should be the mean setting so that an 8-ounce cup will overflow only 0.5% of the time?

A) 6.97 ounces   B) 7.22 ounces   C) 7.34 ounces   D) 7.80 ounces   E) 9.03 ounces

15. The lengths of babies born at Tarth Hospital in the last year were normally distributed. The mean length was 50 cm and the standard deviation was 3.5 cm.

a) What proportion of the babies were shorter than 47.5 cm?

b) What proportion of the babies are longer than 56 cm?

c) What proportion of the babies born at the hospital are between 45 cm and 51 cm?

d) What length would be the cutoff value for the shortest 25% of the babies born last year?

e) Brienne is a baby whose length was 1.77 standard deviations above the mean length. At what percentile was her height in relation to all babies in this distribution?

16. Pygmies live in Zaire (formerly the Belgian Congo) in a region called Ituri (after the river by that name). Colin Turnbull, who lived with a group of pygmies during the early 1950's, wrote a delightful book called The Forest People (1961) telling of his experiences. Turnbull reports that pygmies are less than 4 1/2 feet tall. For each of the questions below assume that the height of pygmies is normally distributed with a mean of 4'3'' and a standard deviation of 2 inches. (Hint: change all units to inches)

   a) Pygmies live in huts made up of a framework of branches covered with leaves. Materials can be gathered and the hut constructed in an afternoon. If the opening is 4 feet high, what proportion of the pygmies will have to duck to enter?

   b) Pygmy culture has very few rules. I will make one up in order to have a question. Suppose only those pygmies who were between 4'2'' and 4'6'' tall were allowed to sing in a molino ceremony. What proportion would be **left out**?

   c) What height would separate the top 10% of the pygmies from the rest of the tribe?

17. A company's manufacturing process uses 500 gallons of water at a time. A "scrubbing" machine then removes most of a chemical pollutant before pumping the water into a nearby lake. Legally the treated water should contain no more than 80 parts per million of the chemical, but the machine isn't perfect and it is costly to operate. Since there's a fine if the discharged water exceeds the legal maximum, the company sets the machine to attain an average of 75 ppm for the batches of water treated. They believe the machine's output can be described by a Normal model with standard deviation 4.2 ppm.

   a) What proportion of the batches of water discharged exceed the 80 ppm standard?

b) The company's lawyers insist that they not have more than 2% of the water discharges over the limit. To what mean value should the company set the scrubbing machine? Assume the standard deviation does not change.

c) Because achieving a mean that low would raise the costs too much, they decide to leave the mean set at 75 ppm and try to reduce the standard deviation to achieve the "only 2% over" goal. Find the new standard deviation needed.

d) Explain what achieving a smaller standard deviation means in this context.

18. Let's say the distribution of the number of hours AP Statistics students sleep the night before a test is normally distributed with a mean of 6.5 and a standard deviation of 0.5.

   a) Sketch the curve with a reasonable scale.

   b) Using the 68-95-99.7 Rule, what proportion of students sleep between 6 and 7 hours before a test?

   c) Using the 68-95-99.7 Rule, 99.7% of students sleep between what hours?

19. Suppose that the scores on the PSAT (Podunk SAT) are approximately normally distributed with a mean of 527.

   a) What is the standard deviation of this distribution if 68% of scores fall between 434 and 620?

   b) What is the standard deviation of this distribution if 95% of scores fall between 434 and 620?

20. The amount of time required for students to finish an AP Statistics test review has a distribution that is approximately normally distributed with a mean of 84 minutes and a standard deviation of 6 minutes.

a) What is the IQR for the distribution of completion times for this test review?
   (Hint: Find the values that separate the top 25% (Q_3) and the bottom 25% (Q_1). Then subtract Q_1 from Q_3.)

b) One common method for finding outliers in a distribution is to look for any values that are more than 1.5 IQRs above the Q3, as well as any values that are 1.5 IQRs below Q1.

   (i) Based on this criteria, how long would a student need to spend on this test review to be considered a high outlier?

   (ii) Based on this criteria, how quickly would a student need to complete this test review to be considered a low outlier?

AP STATISTICS
Review Unit III – Normal Distributions

ANSWERS (make sure you show your work if you want credit!!!)

1. 0, 1, 1

2. Test A ; A: z = 1.8 ; Test B: z = 1.75

3. z = 1.2: This employee's salary is 1.2 standard deviations above the mean salary of all employees at this company.
    z = –2.3: This employee's salary is 2.3 standard deviations below the mean salary for all employees at the company.
Note: Hopefully you did NOT try to relate these z-scores to percentiles or probabilities using the z-table – the problem does NOT state that salaries at this company are normally distributed!

4. B   5. D   6. B   6b. *[well... variance is the square of standard deviation...]*

7. E *(We hope you didn't try to use the z-table for this one!)*   8. C   9. B

10. D   11. A   12.E   13. A   14. A

15   a) 0.2375   b) 0.0432   c) 0.5359   d) 47.64 cm
        e) about the 96th percentile

16.   a) 0.9332   b) 0.3085 + 0.0668 = 0.3753   c) 53.56 inches (roughly 4 feet, 5 1/2 inches)

17.   a) about 11.7%   b) about 71.37 ppm   c) at most 2.43 ppm
        d) The scrubber must be more consistent in its performance from batch to batch.

18.   b) 68%   c) 16%   d) 5 and 8   e) 0.815   f) Andrew is an outlier

19.   Using the empirical rule: a) 93   b) 46.5

20.   a) approximately 8.09 minutes   b) more than 100.19 minutes   c) less than 67.81 minutes