AP Statistics
Random Variables Notes

1) An insurance company offers a "death and disability" policy that pays $10,000 when the policy holder dies (which the company estimates will occur for 1 out of every 1000 people), or $5,000 when the policy holder is permanently disabled (estimated to occur for 2 out of every 1000 people). Based on actuarial information, the company has calculated the probabilities shown in the table below. The company plans to charge $50 per policy. Let the random variable "X" represent the **PROFIT** made by the insurance company per person.

Calculate and interpret the mean (expected value) and standard deviation of "X".

Outcome	Death	Disability	Neither
x	-$9950	-$4950	+$50
P(x)	0.001	0.002	0.997

MEAN:

μ_x or $E(x) = \sum x \cdot P(x)$

$= -9950(0.001) + -4950(0.002) + 50(0.997)$

$\boxed{E(x) = \$30}$

Interpretation: (this is a LONG-RUN average)

If the insurance company takes on a LARGE number of clients, their average PROFIT per client averages out to $30.

STANDARD DEVIATION (and variance)

$Var(x) = \sum (x-\mu)^2 \cdot P(x)$

$= (-9950-30)^2(0.001) + (-4950-30)^2(0.002) + (50-30)^2(0.997)$

$= 149600$ ← (technically, the units on variance are "DOLLARS SQUARED")

σ_x or $SD(x) = \sqrt{Var(x)}$

$= \sqrt{149600} = \boxed{\$386.78}$ ← For a LARGE number of clients, this is the typical/average(ish) difference from the mean profit.

2) Find the mean (expected value) and the standard deviation of the random variable "X".

x	60	70	80	90
P(x)	0.2	0.3	0.4	0.1

Answers: $E(x) = 74$

$SD(x) = 9.165...$

Scaling/Shifting with Means and Variances

remember: variance is (st. dev)^2

$E(X \pm c) =$ $E(X) \pm c$

$Var(X \pm c) =$ $Var(X)$

$SD(X \pm c) =$ $SD(X)$

$E(aX) =$ $a \cdot E(X)$

$Var(aX) =$ $a^2 \cdot Var(X)$

$SD(aX) =$ $a \cdot SD(X)$

For any two random variables, "X" and "Y":

$E(X \pm Y) =$ $E(X) \pm E(Y)$

If "X" and "Y" are independent: ALWAYS a plus!

$Var(X \pm Y) =$ $Var(X) + Var(Y)$

$SD(X \pm Y) =$ $\sqrt{SD(X)^2 + SD(Y)^2}$

"X" and "Y" MUST be independent!!!
If they're NOT, then we cannot determine the variance (or standard deviation) of the combined random variable.

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important for SD on C, D, and F

3. X and Y are two independent random variables with the following attributes:

$E(X) = 11$ $\quad$ $E(Y) = 24$
$SD(X) = 9$ $\quad$ $SD(Y) = 5$

Find the mean and standard deviation of each of these random variables:

a) 3X $\quad E(3X) = 3(11) = \boxed{33}$

$SD(3X) = 3 \cdot 9 = \boxed{27}$

b) Y – 15 $\quad E(Y-15) = 24 - 15 = \boxed{9}$

$SD(Y-15) = \boxed{5}$ $\quad$ ★ shift does NOT affect SD!

c) X + Y $\quad E(X+Y) = 11 + 24 = \boxed{35}$

$SD(X+Y) = \sqrt{9^2 + 5^2} = \boxed{10.296}$

d) X – Y $\quad E(X-Y) = 11 - 24 = \boxed{-13}$

$SD(X-Y) = \sqrt{9^2 + 5^2} = \boxed{10.296}$

ALWAYS ADD VARIANCE!

e) X1 + X2 + X3 $\quad$ (not the same as "3X"!!!)

$E(X_1 + X_2 + X_3) = 11 + 11 + 11 = \boxed{33}$

$SD(X_1 + X_2 + X_3) = \sqrt{9^2 + 9^2 + 9^2}$

$= \sqrt{3} \times 9 = \boxed{15.58}$

f) 5X – 3Y

$E(5X - 3Y) = 5(11) - 3(24)$

$= \boxed{-17}$

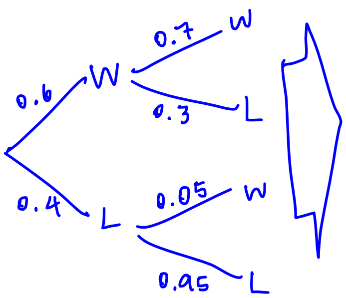
$SD(5X - 3Y) = \sqrt{(5 \times 9)^2 + (3 \times 5)^2}$

$= \boxed{47.43}$

4. **The Podunk Polar Bears** (a football team) have two games left in their season (so far they are winless). Experts estimate that the team has a 60% probability of winning the first game. If they win the first game, they have a 70% chance of winning the 2nd game. Otherwise, they only have a 5% chance of winning the second game.

make a table

Construct a probability model for the number of games that the Polar Bears will win.



0.42

0.18

0.02

ADD

0.38

X = # of wins

X	0	1	2
$P(X)$	0.38	0.20	0.42

The Die (Singular) Game Problem

5. You roll a die. If it comes up a 6, you win \$100.
If not, you get to roll again, and if you get a 6 the second time, you win \$50. If not, you lose ☹
Create a probability model for the amount you win at this game, and find the expected amount you'll win.

X = \$ won

X	100	50	0
$P(X)$	$\frac{1}{6}$	$\frac{5}{36}$	$\frac{25}{36}$
	roll a "6"	$\frac{5}{6}\cdot\frac{1}{6}$	$\frac{5}{6}\cdot\frac{5}{6}$

(if we round)

$E(X) = \$24 \qquad SD(X) = \$38$

Does "$X_1 + X_2$" = "$2X$"? (continuing the Die Game Problem…)

Find the mean and standard deviation of the amount of money won if…

a) we **double** the dollar amounts (and play the game once)

$$E(2X) = 2(24) = \boxed{\$48}$$

$$SD(2X) = 2(38) = \boxed{\$76}$$

b) we **play the game twice** (without doubling the \$ amounts)

Not the same!

$$E(X_1 + X_2) = 24 + 24 = \boxed{\$48}$$

$$SD(X_1 + X_2) = \sqrt{38^2 + 38^2} = \boxed{\$54}$$

c) we **play the game 200 times** (without changing the \$amounts)

$$E(X_1 + X_2 + \dots + X_{200}) = 200(24) = \boxed{\$4800}$$

$$Var(X_1 + X_2 + \dots + X_{200}) = \underbrace{38^2 + 38^2 + \dots + 38^2}_{200 \text{ times}} = 200(38^2)$$

$$SD(X) = \sqrt{200(38^2)} = \boxed{\$537.4\dots}$$

The Bike Store Problem

Bicycles arrive at a bike shop in boxes. The means and standard deviations for the setup phases are given:

Phase	Mean	SD
Unpacking	**4.5**	**0.7**
Assembly	**21.8**	**2.4**
Tuning	**12.3**	**4.7**

Based on past experience, the shop manager makes the following assumptions:

- *the times for the three setup phases are* ***independent***
- *the times for each phase are* ***approximately normally distributed***

a) A customer decides to buy a bike like one of the display models, but wants a different color. The shop has one, still in the box. The manager says they can have it ready in half an hour. Find the probability that they can get the bike setup and ready to go as promised.

$E(U+A+T) = 4.5 + 21.8 + 12.3$

$P(U+A+T \le 30 \text{ min})$?

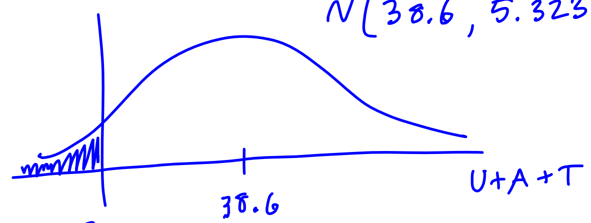
$\mu = 38.6$ min

$SD(U+A+T) = \sqrt{0.7^2 + 2.4^2 + 4.7^2}$

$\sigma = 5.3235\ldots$ min

USE THE NORMAL MODEL:

$N(38.6, 5.3235)$



30 (half-hour or LESS)

$z = \frac{x-\mu}{\sigma}$

$= \frac{30 - 38.6}{5.3235} = -1.6155\ldots$

$P(\text{total time} \le 30 \text{ min})$

$= P(z < -1.615\ldots)$

$= \boxed{0.0531}$

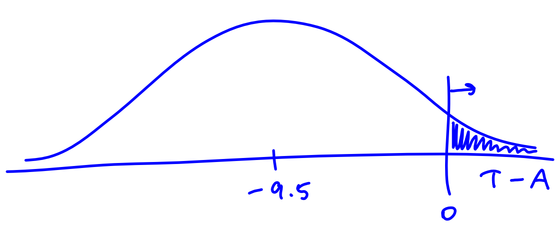
b) What is the probability that the tuning stage takes longer than the assembly stage?

$P(T > A) \ldots$ or $P(T - A > 0)$ → Find μ and σ of $(T-A)$:

$E(T-A) = 12.3 - 21.8 = -9.5$

$SD(T-A) = \sqrt{4.7^2 + 2.4^2} = 5.2773$

$N(-9.5, 5.2773)$



$z = \frac{0 - {}^{-}9.5}{5.2773} = 1.80$

$P(T-A > 0) = \boxed{0.0359}$

The Matchmaker Problem I

In a far-away society, males and females are randomly selected to be matched up with each other for life ☺

Heights	Mean	SD
Males	**69.5**	**3.2**
Females	**65**	**2.8**

We will assume that

- *the heights for adult males and females are* ***independent***
- *the heights of both males and females are* ***approximately normally distributed***

a) Find the probability that the female is paired with a $\underbrace{\text{shorter man}}$.

$M < F$ ... or $M - F < 0$

(This means male is shorter than female)

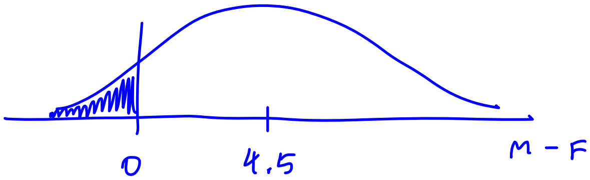
$P(M - F < 0)$

☆ First find μ and σ of $(M-F)$:

$E(M-F) = 69.5 - 65 = 4.5$

$SD(M-F) = \sqrt{3.2^2 + 2.8^2} = 4.252$

Using $N(4.5, 4.252)$:



$z = \frac{x - \mu}{\sigma}$

$= \frac{0 - 4.5}{4.252} = -1.0583$

$= P(z < -1.0583)$

about

$= \boxed{0.1446}$

7. The Matchmaker Problem II

b) Find the probability that the man is at least 12 inches taller than his lady.

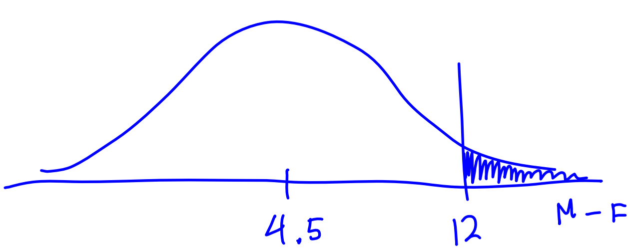
WHOA, NELLY!

this means

$M - F \geq 12$

STILL USING

$N(4.5, 4.252)$ for "$M - F$":



$z = \frac{x - \mu}{\sigma}$

$= \frac{12 - 4.5}{4.252}$

$z = 1.76...$

$P(M - F \geq 12)$

$= P(z > 1.76)$

about

$= \boxed{0.0392}$