AP Statistics Name: NOTES Block: ______
Sampling Distributions with Proportions

Basic steps for probability problems with sampling distributions

1. Define the parameter (using "p" or "μ", NOT "p-hat" or "x-bar"!)
2. Check the necessary conditions
3. Describe the distribution of the statistic (p-hat or x-bar) – Shape, mean and standard deviation.
4. Use the normal model to find the appropriate probability.
5. Interpret the probability in context *(write out a sentence explaining what the probability that you found means)*

The bad apple problem

A large shipment of apples on a truck are to be inspected before being sold in a public market. The inspectors will select a random sample of 150 apples from the truck, and if more than 5% of the apples in the sample are bad, then the entire shipment is rejected. Suppose that in fact 9% of all of the apples on the truck are bad. What is the probability that the shipment is **accepted**?

n p $\hat{p}$

$n = 150$ $p = 0.09$

p = the true proportion of all apples on the truck that are bad.
(or "population")

DESCRIBE THE DISTRIBUTION OF $\hat{p}$:

SHAPE: Approximately normal (since $np \geq 10$ and $nq \geq 10$)

CENTER: $\mu_{\hat{p}} = p = 0.09$

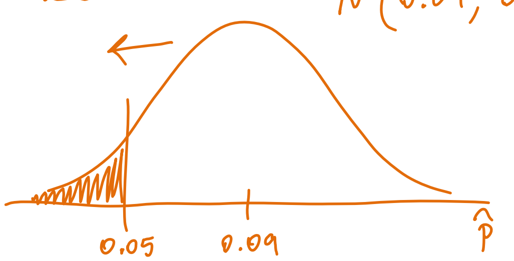
SPREAD: $\sigma_{\hat{p}} = \sqrt{\dfrac{p(1-p)}{n}} = \sqrt{\dfrac{0.09(1-0.09)}{150}} = 0.02337$

CONDITIONS:

- Random sample?
  Yes, they are inspecting a random sample of 150 apples.
  (so the sample is representative of all apples on the truck.)
- 10% condition
  150 apples is reasonably < 10% of all apples in this "large" truck.
  (Therefore, even though we are sampling WITHOUT replacement, this is mathematically similar enough to sampling WITH replacement.)
- Normal approximation
  $np = 150(0.09) = 13.5 \geq 10$ ✓
  $nq = 150(1-0.09) = 136.5 \geq 10$ ✓
  (so we can use the Normal model for $\hat{p}$)

Find $P(\hat{p} < 0.05)$

$N(0.09,\ 0.0234)$



$$z = \frac{\overset{\hat{p}}{\text{obs}} - \overset{p}{\text{mean}}}{\text{st. dev}} = \frac{0.05 - 0.09}{0.023366\ldots} = -1.712$$

st. dev $= \sqrt{\dfrac{pq}{n}}$

$P(\hat{p} < 0.05) = \boxed{0.0435}$

[Interpretation]

Given that 9% of all apples on the truck are bad, the probability that fewer than 5% of a random sample of 150 are bad is about 0.0435.

The lefty-desk problem

Suppose that about 13% of the students at a large college are left-handed. A 200-seat auditorium has been built with 15 "lefty seats." In a class of 90 students, what's the probability that there will **not** be enough seats for the left-handed students? → this means $> \frac{15}{90}$ are lefty… or $\hat{p} > \frac{15}{90}$

extraneous info !!

p = the true proportion of all students at this college who are lefty.

DESCRIBE THE DISTRIBUTION OF $\hat{p}$:

- Roughly normal (since $np \geq 10$ and $nq \geq 10$)
- $\mu_{\hat{p}} = p = 0.13$
- $\sigma_{\hat{p}} = \sqrt{\frac{pq}{n}} = \sqrt{\frac{0.13(0.87)}{90}} = 0.03545$

Conditions:

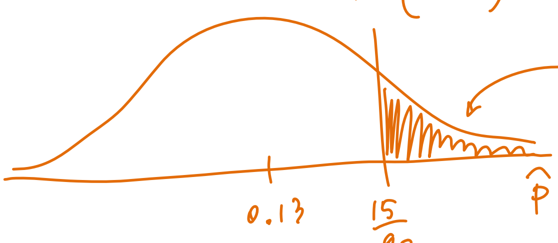
- The 90 students in this class may not be a proper random sample, but (hopefully) are representative of all students (in terms of them being lefty or not)
- 90 students is surely < 10% of all students at this "large college" (therefore even though we are sampling withOUT replacement, it is mathematically similar enough to sampling WITH replacement)
- Normality:

  $np = 90(0.13) = 11.7 \geq 10$ ✓

  $nq = 90(1-0.13) = 78.3 \geq 10$ ✓

(We need to find probability that $\hat{p} > \frac{15}{90}$)

$N(0.13,\ 0.03545)$



(0.16666…)

$z = \frac{\text{obs} - \text{mean}}{\text{st. dev}}$

$= \frac{\frac{15}{90} - 0.13}{0.03545} = 1.0343\ldots$

$P\left(\hat{p} > \frac{15}{90}\right)$

$= P(z > 1.0343)$

$= 1 - 0.8485$ (ish)

$\approx \boxed{0.1515 \text{ (ish)}}$

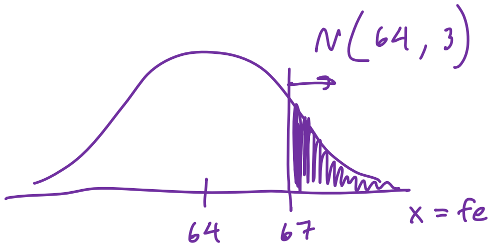
☆ The probability that more than 15 of the 90 students are left-handed is about 0.1515.

AP Statistics Name: NOTES Block: ______
Sampling Distributions with Means

The tall female problem

Adult female heights in the United States are roughly normally distributed with a mean height of about 64 inches, and a standard deviation of about 3 inches.

a) What is the probability that ONE randomly selected female is taller than 67 inches? (we learned these 1st semester)



x = female heights (inches)

$z = \frac{x - \mu}{\sigma}$

$= \frac{67 - 64}{3}$

$z = 1.0$

$P(x > 67)$

$= P(z > 1.0)$

$= 1 - 0.8413 = \boxed{0.1587}$

b) If we take MAAAAAANY random samples of 10 females, describe the distribution of **sample means** for these heights.

(CUSS & BS...)

μ = true mean height for ALL U.S. adult females.

The distribution of $\bar{x}$ will be:

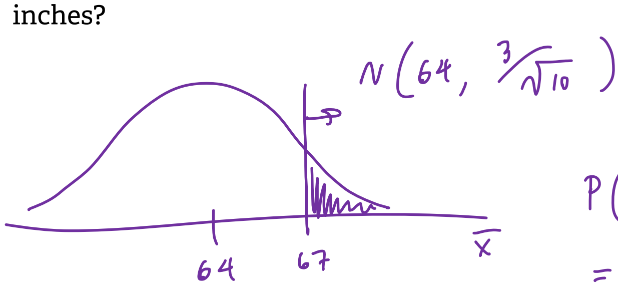
SHAPE: Approximately normal...

CENTER: $\mu_{\bar{x}} = \mu = 64$ inches

SPREAD: $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{3}{\sqrt{10}} = 0.9487$ inches

c) If we take a random sample of 10 females, what is the probability that their **mean height** is greater than 67 inches?

$\bar{x}$



$z = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}}$

$= \frac{67 - 64}{3/\sqrt{10}} = 3.16$

$P(\bar{x} > 67)$

$= P(z > 3.16)$

$= 1 - 0.9992$

$= \boxed{0.0008}$

CONDITIONS

- We have a RANDOM SAMPLE of 10 females...
- ...which is surely < 10% of all female adults in the U.S.
- Population of female heights is normally distributed ✓ (so $\bar{x}$ is also approximately normally distributed)

The rabbit problem

A particular breed of rabbits has a mean weight of 5 pounds, with a standard deviation of 4 pounds. However, the distribution of weights for these rabbits is skewed to the right.

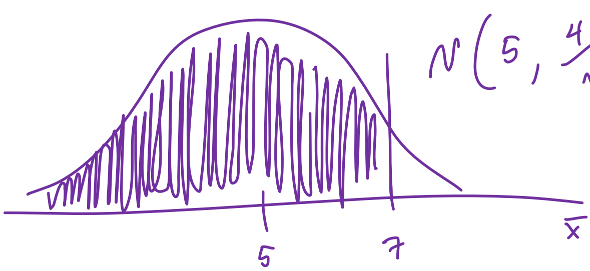
a) If we wish to find the probability that ONE randomly selected rabbit weighs less than 7 pounds, can we calculate this probability using the normal model?

NO. The population of weights is non-normal, so the normal model may not be used here.

b) If we wish to find the probability that a random sample of 35 of these rabbits have a mean weight of less than 7 pounds, can we calculate this probability using the normal model? If so, calculate this probability.

$\bar{x}$

μ = true mean weight for this breed of rabbits. $n = 35$



$N\left(5, \frac{4}{\sqrt{35}}\right)$

$z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}}$

$= \frac{7-5}{4/\sqrt{35}} = 2.958$

$P(\bar{x} < 7)$

$= P(z < 2.96)$

$= \boxed{0.9985}$

CONDITIONS

- Random sample of 35 rabbits...
- ...which is surely < 10% of all rabbits of this breed
- $n > 30$ ✓ (large enough sample)

so we may use the Normal model for the distribution of $\bar{x}$.

c) If we take **ONE SAMPLE** of 80 of these rabbits, what would be the shape of the distribution of weights of the sample?

Since the weights are skewed to the right, the distribution for ONE sample will likely be skewed to the right.

d) Describe the sampling distribution of the sample mean rabbit weights for random samples of 5 rabbits.

Since the sample size is small (< 30ish), the shape will still be SKEWED to the right.

Center: $\mu_{\bar{x}} = 5$ pounds,

spread: $\sigma_{\bar{x}} = \frac{4}{\sqrt{5}} = 1.789$ pounds

e) Describe the sampling distribution of the sample mean rabbit weights for random samples of 80 rabbits.

Since the sample size > 30, the shape will be approximately normal.

center: $\mu_{\bar{x}} = 5$ pounds

$\sigma_{\bar{x}} = \frac{4}{\sqrt{80}} = 0.447$ pounds