AP STATISTICS **Name: ________________________________** **Period: ____**

Probability Models (HW #35)

Show all work and reasoning.

1. X and Y are two independent random variables with the following attributes:

   E(X) = 80 SD(X) = 12
   E(Y) = 10 SD(Y) = 5

   Find the mean and standard deviation of each of these random variables:

   a) X + Y

   b) 3Y – 7

   c) X – Y

   d) $Y_1 + Y_2 + Y_3$

   e) 2X + 4Y

2. **"1000X" vs "$X_1 + X_2 + \ldots + X_{1000}$"** A factory produces a large number of porcelain gnomes for citizens of Podunk who are looking to decorate their lawns. Let the random variable "X" represent the weight of a gnome produced by this factory. The mean of "X" is 18.2 kg, and the standard deviation of "X" is 0.45 kg.

   a) Suppose that we convert the weights of these gnomes from kilograms into grams by multiplying the weight of each gnome by 1,000. Calculate the new mean and standard deviation of the weight of a gnome from this factory when measured in grams. (Note: This is an example of "1000X")

   b) Let's go back to using kilograms. Suppose that we take a random sample of 1,000 gnomes (!) for a very large overseas shipment. Calculate the expected value and standard deviation of the combined weight of 1,000 gnomes when measured in kilograms. (Note: This is an example of "$X_1 + X_2 + X_3 + \ldots + X_{1000}$")

3. A company ships gift baskets that contain apples and oranges. The distributions of weight for the apples, the oranges, and the baskets are each approximately normal. The mean and standard deviation for each distribution is shown in the table below. The weights of the items are assumed to be independent.

Item	Mean	Standard Deviation
Apple	4.5 ounces	0.24 ounce
Orange	5.6 ounces	0.19 ounce
Basket	11.2 ounces	1.77 ounces

a) Let the random variable W represent the total weight of 6 apples. Describe the distribution of W.

b) Let the random variable W represent the total weight of 47 oranges. Describe the distribution of W.

c) Let the random variable W represent the total weight of 4 apples, 9 oranges, and 1 basket. Describe the distribution of W. Then find the probability that W is greater than 83 ounces.

4. In the 4 X 100 medley relay event, four swimmers swim 100 yards, each using a different stroke. A college team preparing for the conference championship looks at the times their swimmers have posted and creates a model based on the following assumptions:

- The swimmers' performances are independent.
- Each swimmer's times follow a Normal model.
- The means and standard deviations of the times (in seconds) are as shown:

Swimmer	**Mean**	**SD**
Backstroke	50.72	0.24
Breaststroke	55.51	0.22
Butterfly	49.43	0.25
Freestyle	44.91	0.21

a) What are the mean and standard deviation for the relay team's total time in this event?

b) The team's best time so far this season is 3:19.48 (that's 199.48 seconds). What is the probability that the team will beat its own best time at the conference championship?

5. A local gourmet donut stand sells a plethora of delicious specialty donuts to folks in downtown Austin. Customers can choose to add extra toppings (such as fruits, nuts, candy, chocolate syrup, etc.) for an additional charge.

A business associate has determined that the following probability model defines the amount of money that one customer might spend at their donut stand.

X ($ spent)	$4.50	$5.50	$6.50	$7.50	$8.00 *(the max)*
P(x)	0.13	0.45	0.27	0.09	0.06

a) What are the mean (a.k.a., expected value) and the standard deviation for the amount of money spent by a customer at this donut stand?

b) Let us assume that during a Saturday afternoon, the donut stand gets 50 customers in a busy hour. The owner of the donut stand has to determine whether they will be able to make enough money to pay "rent" to the city, and would like to make at least $320 during this busy hour. How likely is it that the stand makes a total of at least $320 from a sample of 50 customers?

6. A slot machine at a casino costs $2.00 to play. It is designed such that each play of the game then has a mean payout amount of $1.77, with a standard deviation of $87. Let us suppose that the slot machine is played exactly 800 times in a day.

a) If the machine is played 800 times in a day, what are the mean and standard deviation of the casino's total **profits** from this slot machine for that day?

b) Based on anecdotal (casual) observations, the casino owner is concerned that a lot of people seem to be winning the jackpot on this slot machine. With a large enough sample size, the casino's total profits for 800 plays of the slot machine are approximately normally distributed. Based on the mean and standard deviation calculated in part (a), what is the probability that this slot machine will lose the casino more than $5,000 in a given day?
(In other words, what is the probability that this slot machine's profits are below -5,000?)

c) Suppose that on a given day, this slot machine loses the casino more than $5,000. Based on your answer to part (b), would you consider this to be a **RARE** or **UNLIKELY** occurrence? Explain.

7. Each full carton of Grade A eggs consists of 1 randomly selected empty cardboard container and 12 randomly selected eggs.

The weights of the empty cardboard containers, *C*, have a mean of 20 grams and a standard deviation of 1.7 grams.

The weights of the individual Grade A eggs , *E*, have a mean of 68.3 grams and a standard deviation of 2.23 grams.

It is reasonable to assume independence between the weights of the empty cardboard containers and the weights of the eggs. It is also reasonable to assume independence among the weights of the 12 eggs that are randomly selected for a full carton.

Let the random variable *X* represent the weight (in grams) of a full carton of Grade A eggs (empty cardboard container **plus** 12 randomly selected eggs).

a) What is the mean of X ?

b) What is the standard deviation of *X* ?

8. A mathematics competition uses the following scoring procedure to discourage students from guessing (choosing an answer randomly) on the multiple-choice questions. For each correct response, the score is 7. For each question left unanswered, the score is 2. For each incorrect response, the score is 0. If there are 5 choices for each question, what is the minimum number of choices that the student must eliminate before it is advantageous to guess among the rest?