

## Matched Pairs vs. Two Independent Samples

For problems 1 – 4, name the appropriate inference procedure, and write the hypotheses.

DO NOT ACTUALLY WORK OUT THESE FIRST 4 PROBLEMS.

1. **Bacteria 1** Investigators at the U.S. Department of Agriculture wished to compare methods of determining the level of bacterial contamination in beef. Two different methods (A and B) of determining the level of contamination were used on each of eight randomly selected specimens of a certain type of beef. The data obtained, in millimicrobes/liter of ground beef, for each of the methods are shown in the table below.

		Specimen							
		1	2	3	4	5	6	7	8
Method	A	22.7	23.6	24.0	27.1	27.4	27.8	34.4	35.2
	B	23.0	23.1	23.7	26.5	26.6	27.1	33.2	35.0

Is there a significant difference in the mean amount of bacteria detected by the two methods for this type of beef? Provide a statistical justification at the  $\alpha = 0.05$  level to support your answer.

2. **Bacteria 2** Investigators at the U.S. Department of Agriculture wished to compare methods of determining the level of bacterial contamination in beef. Two different methods (A and B) of determining the level of contamination were tested. Sixteen randomly selected specimens of a certain type of beef were used in this study – eight of them were randomly assigned to be tested with method A, and the remaining eight were tested with method B. The data obtained, in millimicrobes/liter of ground beef, for each of the methods are shown in the table below.

Method	A	22.7	23.6	24.0	27.1	27.4	27.8	34.4	35.2
	B	23.0	23.1	23.7	26.5	26.6	27.1	33.2	35.0

Is there a significant difference in the mean amount of bacteria detected by the two methods for this type of beef? Provide a statistical justification at the  $\alpha = 0.05$  level to support your answer.

3. **Homework 1** Mrs. Donald wants to know if the homework load is different in the English and mathematics department at Westward High School. She randomly selects 24 students. Twelve record the number of hours they spend during the semester on English homework and the other twelve record the number of hours they spend during the semester on math homework. Is there evidence at the 5% level of significance that students spend more time on math homework, on average, than English homework?

English	54	125	56	72	48	91	55	111	97	90	75	84
Math	88	89	91	112	63	90	55	98	120	131	95	104

4. **Homework 2** Mrs. Donald wants to know if the homework load is different in the English and mathematics department at Westward High School. She randomly selects 12 students who agree to record the number of hours they spend on homework in the two disciplines for the semester. Is there evidence at the 5% level of significance that students spend more time on math homework, on average, than English homework?

	A	B	C	D	E	F	G	H	I	J	K	L
English	54	125	56	72	48	91	55	111	97	90	75	84
Math	88	89	91	112	63	90	55	98	120	131	95	104

**Work out the remaining problems in this packet COMPLETELY.**

5. **Test Prep 1** The managers of a popular test preparation company are developing two different preparation programs for an important exam that college students must take before applying to graduate school. 20 students have volunteered to participate in one of the company's prep programs. All of these students have already taken the exam once, however they hope to improve their score after completing the prep course. Half of the 20 students are randomly assigned to prep course "A", while the remaining students are assigned to prep course "B". After the course, the students all take the exam again, and the tables below show the difference (after – before) in each student's exam scores.

Prep Course "A"	2	-1	12	2	9	4	8	0	1	7
Prep Course "B"	15	17	-3	4	27	5	11	13	-4	10

For the type of students who would volunteer for this company's prep courses, construct and interpret a 95% confidence interval for the difference in mean score improvement between prep course "A" and prep course "B". (Note: You may wish to compare this problem to the next problem, BEFORE you start writing anything...)

6. **Test Prep 2** The same test preparation company is now working on a third preparation program for the same graduate school entrance exam. From a large number of students that are signed up to take one of the company's prep courses, 10 students are randomly selected for this new course. All of these students have already taken the exam once, however they hope to improve their score after completing the prep course. After completing the course, the students all take the exam again, and the tables below show the "before" and "after" scores for each of the ten students.

	Student									
	A	B	C	D	E	F	G	H	I	J
Before prep course	18	18	21	18	18	20	23	23	21	17
After prep course	24	25	33	29	33	36	34	36	34	27

For the type of students who would sign up for this company's prep courses, construct and interpret a 95% confidence interval for the difference in mean scores (after – before) on this graduate school entrance exam.

7. **Acid Rain** An environmental agency is investigating whether industrial pollution is leading to an increase in the acidity of rainfall in a large rural county. Researchers collect samples of soil from ten randomly selected locations in the county, and measure the pH levels from both the surface and the subsurface soil at each of the locations. The researchers are hypothesizing that there will be greater acidity near the surface – thus a lower pH value – than in the subsurface soil. The results from the analysis are shown below.

Location	I	II	III	IV	V	VI	VII	VIII	IX	X
Surface pH level	7.85	7.73	7.85	5.40	7.35	6.17	5.92	6.18	6.43	5.68
Subsurface pH level	8.52	8.01	7.99	6.93	7.21	6.91	6.10	7.26	7.18	5.88

- a) Do the data from this study provide evidence of lower mean pH levels for soil near the surface than in the subsurface in this large rural county? Perform a test at the 1% level of significance.

- b) The above test was conducted at the 1% level of significance, and the alternative hypothesis was one-sided (hopefully you performed a 1-sided test?). Thus if you were to perform the same test by using a confidence interval, the corresponding confidence level is 98%. Using the same data as you used in part (a), quickly calculate the 98% confidence interval using your calculator (no need to show work or conditions).

Does this interval agree with the conclusion to your test in part (a)? Explain.

8. **Salaries** A researcher conducted a study to investigate whether local tech companies tend to offer higher salaries to males than to females. Using information from the county employment records, the researcher randomly selected one male and one female from among everyone who had accepted an entry-level position at a company (the male and female were selected from the same position, with similar responsibilities and necessary qualifications, etc). This process was repeated for a total of 8 randomly selected local technology companies. The salaries are shown in the table below.

Company	1	2	3	4	5	6	7	8
Men	\$42,100	\$47,400	\$52,800	\$32,700	\$57,500	\$41,500	\$79,800	\$89,800
Women	\$39,500	\$48,500	\$51,900	\$33,400	\$50,500	\$40,200	\$75,100	\$82,600

- a) At  $\alpha = 0.05$ , do the data provide convincing evidence that, on average, women receive lower salaries than men for similar entry-level positions at technology companies?
- b) A law firm is interested in representing a group of women who wish to file a lawsuit against some of these companies for gender discrimination. However if they do so, the law firm would need to spend a significant amount of time and money. Thus they will only agree to represent these women if there is statistical evidence that on average, women receive lower salaries than men at these technology companies. Otherwise, the law firm believes it is unlikely that they will be able to recover their expenses.

Based on your conclusion in part (a), what type of error (type I or type II) is possible? Describe a consequence of this error as it relates to the law firm's decision whether to carry forward with this lawsuit.

9. **Sleep** W. S. Gosset studies whether the herb *laevohyoscyamine hydrobromide* can increase the mean amount of sleep that people get at night. Gosset records the amount of sleep received by 8 patients on two different nights – once with the herb, and once without the herb (the night on which each patient received the herb was randomized). He conducts a test of significance at  $\alpha = 0.02$  on the following hypotheses:

$\mu_d$  = mean increase in hours of sleep using the herb *laevohyoscyamine hydrobromide*

$$H_0: \mu_d = 0 \quad H_A: \mu_d > 0$$

- a) A  $t$ -test on these hypotheses has a  $t$ -statistic of 2.323. Find the  $P$ -value, and state the conclusion regarding the hypotheses.

Hint: To find the  $P$ -value, you can use “tcdf” on your calculator. You can also use the  $t$ -table to get a range of  $P$ -values.

- b) This conclusion, of course, may be incorrect. If so, which type of error was made?
- c) Suppose that Gosset wishes to use a  $t$ -interval to test the same hypotheses stated above. Given that he wishes to conduct the test at the 2% level of significance, what would be the appropriate confidence level?
- d) Suppose that the use of the herb *laevohyoscyamine hydrobromide* actually helps people get one extra hour of sleep each night, on average, and that Gosset estimates that the power of the test is 0.09. Interpret the value of 0.09 in the context of the study.
- e) How would the power of the test change if Gosset had included 40 patients instead of 13?
- f) How would the power of the test change if Gosset had conducted the test at  $\alpha = 0.05$  instead of 0.02?