Algebra II

Section 6-5: Theorems about Roots of Polynomials
Section 6-6: Fundamental Theorem of Algebra

Review of Polynomial Division:

Recall that if a synthetic division gives you a remainder of 0, that means the x-value is a root/zero:

Determine whether $x + 4$ is a factor of each polynomial.

a. $x^2 + 6x + 8$

The synthetic division gives a remainder of zero – therefore:
- (x + 4) is a factor of the polynomial, and
- x = -4 is a zero of the function.

But when looking for roots, how do we know what numbers to do synthetic division with? For this, we can use the Rational Root Theorem.

Rational Root Theorem

Theorem — **Rational Root Theorem**

If $\frac{p}{q}$ is in simplest form and is a rational root of the polynomial equation $a_nx^n + a_{n-1}x^{n-1} + \ldots + a_1x + a_0 = 0$ with integer coefficients, then p must be a factor of a_0 and q must be a factor of a_n.

You can use the Rational Root Theorem to find any rational roots of a polynomial equation with integer coefficients.

1 EXAMPLE Finding Rational Roots

Find the rational roots of $x^3 + x^2 - 3x - 3 = 0$.

The only rational root of $x^3 + x^2 - 3x - 3 = 0$ is -1

2 EXAMPLE Using the Rational Root Theorem

Find the roots of $2x^3 - x^2 + 2x - 1 = 0$.

Irrational Root Theorem and Imaginary Root Theorem

Theorem — **Irrational Root Theorem**

Let a and b be rational numbers and let $\sqrt{b}$ be an irrational number. If $a + \sqrt{b}$ is a root of a polynomial equation with rational coefficients, then the conjugate $a - \sqrt{b}$ also is a root.

Theorem — **Imaginary Root Theorem**

If the imaginary number $a + bi$ is a root of a polynomial equation with real coefficients, then the conjugate $a - bi$ also is a root.

3 EXAMPLE Finding Irrational Roots

A polynomial equation with integer coefficients has the roots $1 + \sqrt{3}$ and $-\sqrt{11}$. Find two additional roots.

4 EXAMPLE Finding Imaginary Roots

A polynomial equation with integer coefficients has the roots $3 - i$ and $2i$. Find two additional roots.

Find a third-degree polynomial equation with rational coefficients that has the given roots:

19. 1 and $3i$

Fundamental Theorem of Algebra

Theorem Fundamental Theorem of Algebra

If $P(x)$ is a polynomial of degree $n \geq 1$ with complex coefficients, then $P(x) = 0$ has at least one complex root.

Corollary

Including complex roots and multiple roots, an nth degree polynomial equation has exactly n roots; the related polynomial function has exactly n zeros.

1 EXAMPLE Using the Fundamental Theorem of Algebra

For the equation $x^3 + 2x^2 - 4x - 6 = 0$, find the number of complex roots, the possible number of real roots, and the possible rational roots.

You can often find all the zeros of a polynomial function by using some combination of graphing, the Factor Theorem, polynomial division, the Remainder Theorem, and the Quadratic Formula.

2 EXAMPLE Finding All Zeros of a Polynomial Function

Find all the zeros of $f(x) = x^3 + x^2 - x + 2$.