AP Review IV - Hypothesis Tests/Confidence Intervals (30% – 40%)

IMPORTANT: All tests assume a simple random sample *from the population being studied*.
All tests are testing to see if the **sample** has enough evidence to claim that the
populations involved meet the criteria of the problem.

TO RECEIVE FULL CREDIT FOR A HYPOTHESIS TEST YOU MUST:
    1) write the null and alternative hypothesis and define each variable
    2) write which test you are using in words or with the appropriate formula and why you chose that test
    3) write and check all conditions for that test
    4) give the test statistic and the p-value (or critical value if you prefer) and df , if applicable
    5) reject or fail to reject Ho based on the p-value (or critical value)
    6) write a conclusion in terms of the problem
You either have enough evidence to claim whatever the alternative hypothesis represents (reject Ho)or you
do not have enough evidence to claim whatever the alternative hypothesis represents (fail to reject Ho)

TO RECEIVE FULL CREDIT FOR A CONFIDENCE INTERVAL YOU MUST:
    1) correctly identify the type of interval by name or formula
    2) write and check all conditions for that interval
    3) correctly calculate the interval – show work
    4) correctly interpret the interval in terms of the problem
    5) you may also be required to correctly interpret the confidence level
NOTE: All confidence intervals have the same assumptions of the corresponding hypothesis test.
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Type I errors, Type II errors, and Power

Type I error – (α) – when Ho is true but you go with Ha.
Type II error – (β) – when the alternative, Ha, is true but you go with Ho
Type I and Type II errors are inversely related; as one increases the other decreases.

Power = 1 – β, so Type II errors and power are inversely related. Type I errors and power
        are directly related.

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Questions to ask before beginning a hypothesis test or confidence interval

1) Are the data categorical or quantitative?

2.) How many samples?

3) Are the data independent or dependent?

4) What **exactly** are we trying to learn? About which populations do we wish to make an inference?

5) What are the conditions (assumptions) and how do we check them?

Notes for Inference

Type of test	Null hypothesis	conditions	checks
Proportions			
One Sample	H_0: P_1 = a proportion	1. SRS from population 2. large enough sample	1. read the prompt 2. np and n(1–p) ≥ 10
Two Sample	H_0: P_1 = P_2	1.SRS from both populations or random assignment 2. large enough samples	1. read the prompt 2. np and n(1–p) ≥10 for both populations
Means			
One Sample	H_0: μ = # (df = n–1)	1. SRS from population 2. independent individuals 3. population is normal	1. read the prompt 2. are they? 3. n ≥ 30 (CLT) or graph unimodal and symmetric
Two Sample	H_0: μ_1 = μ_2 (df from calculator or n–1 from smallest n)	1.SRS from both populations or random assignment 2. samples are independent 3. both populations are normal	1. read the prompt 2. are they? 3. both graphs are unimodal and symmetric
Matched Pairs	H_0: μ_d = 0 (df = n–1)	1. SRS from population 2. data are dependent 3. population of **differences is normal**	1. read the prompt 2. are they? 3. graph of differences (L_3) is unimodal and symmetric
Chi-Square			
Goodness of Fit	H_0: data does fit the hypothesized proportions (be specific) (df = # of cells – 1)	1. SRS from the population 2. large enough sample	1. read the prompt 2. all **expected** counts ≥ 5
Homnogeneity (several groups – one variable)	H_0: proportions are the same for all populations df = (r–1)(c–1)	1.independent random counts 2. large enough samples	1. read the prompt 2. all **expected** counts ≥ 5
Independence (one population classified on two variables)	H_0:The two variables are independent df = (r–1)(c–1)	1. SRS from the population 2. large enough sample	1. read the prompt 2. all **expected** counts ≥ 5
Regression			
For slope	H_0: β = 0	1. independent SRS from population being studied 2.Scatterplot appears linear 3. The distribution of y values at any given x value is normal 4. The distributions all y values have the same standard deviation	1. read the prompt 2. does it? Check residual plot 3. graph residual plot 4. variability of points does not appear to be changing with x

Note: For chi-squared, we write for H_0: There is **no** relationship between the two variables.
Note: If the sample is less than 10% of the population, independence may become a moot point.

For each problem below, identify the test or procedure you would use. ~~Write and check all assumptions.~~ If the problem requires you to find the sample size, find the sample size.

1. Let's say that 58% of 462 randomly selected college freshman reported being overwhelmed by the task of managing their own time. (No one is there to nag them about allowing adequate time to study, do laundry, sleep, or get to class.) Construct and interpret a 90% confidence interval for π, the true proportion of college freshman that are overwhelmed by the task of managing their own time. (pjs)

2. A psychologist finds that fidgety patients tap their fingers on average 500 times during a 2 hour period. You wish to test this claim for the same patients, but in a room decorated with soothing colors and soft lighting. You believe that a warmly decorated room may alter the fidgeting behavior, causing subjects to tap their fingers less than they do under normal conditions. A random sample of 400 patients has a mean of 420 taps and a standard deviation of 60 taps in a 2 hour period. Test the claim at the 0.05 significance level. (pjs)

3. What would change in your procedure for the above problem if the random sample contained only 10 patients? What additional information would you need to meet your assumptions?

4. If a random sample of 1,000 Austin residents contain 535 persons who prefer Time Warner internet to AT&T internet, is this sufficient evidence to conclude that more than half the people in Austin prefer Time Warner at the 0.01 level?

5. To estimate the number of young animals per herd of mouse lemurs, a biologist randomly selects sites in a region of Madagascar and counts the young members of herds sighted near the chosen location. The mean from 25 sites is 8.2, with a standard deviation of 3.4. If the count of offspring is normally distributed, find an 80% confidence interval for the mean number of young per herd.

6. Smeltzer wants to know how many hours per week senior students spend with their friends in person (not texting or phoning). How many senior students must be randomly selected if she wants a 90% confidence level with a margin of error of no more than 1.5 hours. Previous studies have had a standard deviation of 4 hours.

7. A sample of 481 historians responded to questions about the performance of various U.S. presidents, and the results were presented at the annual conference of the Organization of American Historians (Associated Press, March 28, 1991). Of the 481 surveyed, 433 responded that Ronald Reagan lacked the proper intellect for the presidency. Construct a 90% confidence interval for the true proportion of all historians who believe that Reagan lacked the proper intellect for the presidency. (Note: We are assuming that the 481 historians were chosen randomly.)

8. A consumer group is interested in estimating the proportion of over ripe peaches at a local H.E.B. grocery store. How many randomly selected products should be checked in order to estimate this proportion within 3% with 96% confidence?

9. Mrs. Donald wants to know if the homework load is different in the English and mathematics department at Westwood High School. She randomly selects 24 students. Twelve record the number of hours they spend during the semester on English homework and the other twelve record the number of hours they spend during the semester on math homework. Does there appear to be a significant difference in the homework load between the two departments?

English	54	125	56	72	48	91	55	111	97	90	75	84
Math	88	89	91	112	63	90	55	98	120	131	95	104

10. Mrs. Donald wants to know if the homework load is different in the English and mathematics department at Westwood High School. She randomly selects 12 students who agree to record the number of hours they spend on homework in the two disciplines for the semester. Does there appear to be a significant difference in the homework load between the two departments?

subject	A	B	C	D	E	F	G	H	I	J	K	L
English	54	125	56	72	48	91	55	111	97	90	75	84
Math	88	89	91	112	63	90	55	98	120	131	95	104

11. To test the effectiveness of a new drug for insomnia, 80 patients at a clinic were given a pill containing the drug and 80 others were given a placebo. At the 0.01 level of significance, what can we conclude about the effectiveness of the drug if in the first group 56 of the patients felt a beneficial effect while 38 of those who received a placebo felt a beneficial effect? (The mind is a curious thing!)

12. Is there a correlation between test anxiety and exam score performance? Data on x = score on a measure of test anxiety and y = exam score consistent with summary quantities given in the paper "Effects of Humor on Test Anxiety and Performance" (*psych. Reports* (1999): 1203-1212) appears below. Test to see if the linear regression equation is useful in predicting test score using anxiety score.

x	23	14	14	0	7	20	20	15	21
y	43	59	48	77	50	52	46	51	51

13. A student survey is designed to determine if there is a difference between freshman and senior relationships. The proportion of students in each class who have been "going steady" for 3 months or longer was of interest. Of the 213 freshman surveyed, 20 had been going steady for at least 3 months. Of the 154 seniors surveyed, 21 had been going steady for at least 3 months. Construct a 98% confidence interval for the difference between the two proportions.

14. A traffic study interested in color preferences asks 133 randomly selected drivers which of the three colors (yellow, red, green) they least like. Does there seem to be a gender difference in color preference?

	red	green	yellow
male	27	14	26
female	28	18	20

2002B #6

In September 1990, each student in a random sample of 200 biology majors at a large university was asked how many lab classes he or she was enrolled in. The sample results are shown below.

Number of Lab Classes	Number of Students
0	28
1	62
2	58
3	28
4	16
5	8
(Total)	200

$\bar{x} = 1.83$
$s = 1.29$

To determine whether the distribution has changed over the past 10 years, a similar survey was conducted in September 2000 by selecting a random sample of 200 biology majors. Results from the year 2000 sample are shown below.

Number of Lab Classes	Number of Students
0	20
1	72
2	60
3	10
4	26
5	12
(Total)	200

$\bar{x} = 1.93$
$s = 1.37$

a) Do the data provide evidence that the mean number of lab classes taken by biology majors in September 2000 was different from the mean number of lab classes taken in 1990? Perform an appropriate statistical test using $\alpha = 0.10$ to answer this question.

b) Does the test in (a) address the question of whether the distribution of number of lab classes was different in 2000 than it was in 1990? If so, explain your reasoning. If not, carry out an appropriate statistical test using $\alpha = 0.10$ to answer this question.

c) Use the results of your analyses in (a) and (b) to write a few sentences that summarize how the distribution of the number of lab classes did or did not differ. Use appropriate graphs to help communicate your message. This summary should be understandable to someone who has not studied statistics.

In order to monitor the populations of birds of a particular species on two islands, the following procedure was implemented.

Researchers captured an initial sample of 200 birds of the species on Island A; they attached leg bands to each of the birds, and then released the birds. Similarly, a sample of 250 birds of the same species on Island B was captured, banded, and released. Sufficient time was allowed for the birds to return to their normal routine and location.

Subsequent samples of birds of the species of interest were then taken from each island. The number of birds captured and the number of birds with leg bands were recorded. The results are summarized in the following table.

	Island A	Island B
Number captured in subsequent sample	180	220
Number with leg bands in subsequent sample	12	35

Assume that both the initial sample and the subsequent samples that were taken on each island can be regarded as random samples from the population of birds of this species.

a) Do the data from the subsequent samples indicate that there is a difference in proportions of the banded birds on these two islands? Give statistical evidence to support your answer.

b) Researchers can estimate the total number of birds of this species on an island by using information on the number of birds in the initial sample and the proportion of banded birds in the subsequent sample. Use this information to estimate the total number of birds of this species on Island A. Show your work.

c) The analyses in parts (a) and (b) assume that the samples of birds captured in both the initial and subsequent samples can be regarded as random samples of the population of birds of this species that live on the respective islands. This is a common assumption made by wildlife researchers. Describe two concerns that should be addressed before making this assumption.